CONSEIL INTERNATIONAL DES UNIONS SCIENTIFIQUES
INTERNATIONAL COUNCIL OF SCIENTIFIC UNIONS

UNION GÉODÉSIQUE ET GÉOPHYSIQUE INTERNATIONALE
INTERNATIONAL UNION OF GEODESY AND GEOPHYSICS

Bulletin de l'Association Internationale d'Hydrologie Scientifique

N° 5

MARS 1957

Published on behalf of
THE INTERNATIONAL ASSOCIATION OF SCIENTIFIC HYDROLOGY
by
CEUTERICK
66, RUE VITAL DECOSTER
LOUVAIN (Belgium)

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LES PROPOS DU SECRÉTAIRE

1) Notre bulletin n'était jusqu'à présent qu'un tiré à part du Bulletin d'information de l'UGGI.

Nous avions accepté cette façon de faire, espérant pouvoir réduire les frais de publication. L'expérience a fait apparaître certains inconvénients du système et notamment la difficulté de publier dans un bulletin s'adressant à l'Union de multiples et longs exposés dont l'intérêt était limité aux membres de l'Association d'Hydrologie Scientifique.

Le secrétaire général de l'Union, Mr Laclavère, a très bien compris ces difficultés et c'est avec son accord que nous aurons dès 1957 un bulletin indépendant dont ces lignes préfacent le premier numéro.

Comme on l'a décidé à Rome, ce bulletin présentera une partie administrative et une autre réservée aux publications scientifiques. Les charges financières nous obligeront sans doute parfois à limiter la partie scientifique : nous ferons de notre mieux à cet égard. Nous n'avons pas à revenir sur les raisons de la nécessité de la partie dite administrative. Elle serait justifiée rien que par la nécessité de vous tenir au courant de la vie des Associations et Comités qui ont des rapports avec nous, tout en n'appartenant pas à notre Union. L'organisation de nos Assemblées et des Symposia sera d'ailleurs facilitée par les communications que nous vous adresserons.

2) Dans ces propos du secrétaire, vous nous permettrez lors de la parution de chaque numéro, de vous présenter en quelques mots, des communications, des avis, des notes de tous genres qu'il est impossible de mettre sous un titre autre que celui de « propos ». Nous vous demanderons d'y prêter quelque attention si vous voulez vous tenir au courant de la vie de l'Association. Nous nous permettrons au cours de ces propos, de vous parler assez familièrement, comme dans une conversation.

3) Nous profitons immédiatement de cette liberté que nous nous accordons pour vous demander de nous conseiller sur le point suivant : l'Union, dans sa politique actuelle, pousse à l'organisation par les Associations de Symposia, dans le genre des Symposia Darcy à Dijon, en dehors des Assemblées Générales. Ces Symposia attirent l'attention sur nos travaux dans des contrées qui ne connaissent guère nos Assemblées par suite de la limitation du nombre de celles-ci.

L'UNESCO se montre d'ailleurs favorable à l'organisation de ces Symposia, mais le secrétaire-trésorier de l'Association voit ses finances menacées par le coût de la publication des mémoires présentés à ces Symposia. La générosité de l'UNESCO s'attache surtout à favoriser les déplacements des participants. Or, l'organisation et les publications du Symposia de Dijon nous coûteront environ 6000 dollars ! Le Conseil, réuni à Dijon, a bien voulu accepter notre proposition de ne faire aucune distribution gratuite de ces « Transactions », mais en dépit des efforts de certains adhérents dont je voudrais parler ci-dessous, l'opération « Dijon » restera vraisemblablement déficitaire au point de vue financier, bien que les lettres que nous avons reçues de partout en font une opération des plus heureuses au point de vue scientifique.

4) Je viens de dire un mot de ceux d'entre vous qui se sont dévoués pour assurer la diffusion des Comptes-Rendus des Symposia de Dijon. Nous ne pouvons nous permettre de citer des noms, mais nous adressons les remerciements de l'Association à tous ceux qui ont fait connaître ces publications, notamment par des articles appropriés dans leurs revues nationales.

A Rome et à Dijon, nous avons maintes fois répété : on ne connaît pas suffisamment notre travail. Nous vous demandons encore une fois votre aide à tous. Nous avons l'impression que de courts comptes-rendus dans les journaux scientifiques nationaux ont un rendement des plus favorables sur la vente.

Nous nous excusons de ces considérations bien terre à terre : nous ne pensions pas avoir une âme de commerçant, mais nous apercevons plus que jamais que nous ne pouvons vivre qu'avec les deux pieds sur terre.

ASSEMBLÉE DE TORONTO

1) Nous avions fixé le 1er avril comme date limite de la rentrée des titres des communications qui seront présentées. Nous l'avons fait par avis personnel aux représentants nationaux qui nous ont été désignés. Nous l'avions rappelé dans le numéro du 1er Décembre 1956 du Bulletin. Mais au moment où nous rédigeons cet avis le numéro en question n'est pas encore sorti de presse.

Nous craignons donc que certains pays aient oublié la date limite. Nous accepterons donc les relevés de titres jusqu'au 30 avril prochain.

2) Nous demandons au représentant pour l'hydrologie de chaque pays de ne pas oublier la date du 1er juin 1957 pour la rentrée des textes des rapports. Pouvons-nous insister pour que les auteurs n'oublient pas de joindre à leur rapport un résumé comportant au maximum une page dactylographiée avec double interligne. Si c'est possible, le représentant national nous enverra ces résumés aussitôt que possible, même avant le texte des rapports.

3) A titre purement indicatif, nous donnons ci-dessous un premier essai du programme, en insistant beaucoup sur le mot *essai*.

Mardi 3 septembre

Avant-midi : Assemblée Générale de l'Union. Convocation Hall.
Après-midi : Séance du Conseil de l'Association.

Mercredi 4 septembre

Avant-midi : Séance commune avec les Associations d'Océanographie et de Météorologie : Thème général : Bilan Hydrologique.
L'Association d'Hydrologie interviendra en présentant celles des communications se rattachant à ce thème et vraisemblablement celles sur l'influence de la végétation.
Après-midi : Idem.

Jeudi 5 septembre

Avant-midi : Com. Neiges et Glaces : 1re Séance.
Com. Eaux souterraines : 1re Séance.
Après-midi : Assemblée générale. Adresse présidentielle. Rapport du Secrétaire. Exposé de diverses questions et discussions.

Vendredi 6 septembre

Avant-midi : Com. Neiges et Glaces : 2me Séance.
Com. Eaux souterraines : 2me Séance.
Après-midi : Com. Neiges et Glaces : 3me Séance.
Com. Eaux souterraines : 3me Séance.

Samedi 7 septembre

Avant-midi : Com. Neiges et Glaces : 4me Séance.
Com. Eaux souterraines : 4me Séance.

Dimanche 8 septembre

Excursion.

Lundi 9 septembre

Avant-midi : Com. Neiges et glaces : 5me Séance.
Com. Eaux souterraines : 5me Séance.
Après-midi : Com. Erosion Continentale : 1re Séance.
Comité des Mesures et Instruments.

Mardi 10 septembre

Avant-midi : Com. Erosion Continentale : 2me Séance
Comité des Précipitations : 1re Séance
Après-midi : Com. Erosion Continentale : 3me Séance
Com. des Eaux de Surface : 1re Séance (Evaporation)

Mercredi 11 septembre

Avant-midi : Com. Eaux de surface : 2me Séance (Evaporation)
Comité des Neiges et Glaces : 6me Séance
Après-midi : Com. Eaux de surface : 3me Séance
Comité des Neiges et Glaces : 7me Séance

Jeudi 12 septembre

Avant-midi : Rosée, condensations et précipitations occultes (1re Séance)
Après-midi : Rosée, condensations et précipitations occultes (2me Séance)
Couuité des précipitations : 2e Séance

Vendredi 13 septembre

Avant-midi : 9 h. Com. des Eaux de surface (4me Séance)
à 11 h. Discussion sur la Standardisation des caractéristique hydrologiques.
Après-midi : 14.30 h. Com. Eaux de surface : 5me Séance.
17 h. Assemblée générale.

Samedi 14 septembre

Avant-midi : Assemblée de clôture de l'Union.

THE SECRETARY'S PROPOSALS

1) Up to the present our Bulletin has been printed only as part of the Information Bulletin of UGGI. We have accepted this method in the hope that publication costs would be reduced. Experience has shewn certain difficulties particularly the publication in the Union's Bulletin of a large number of papers of limited interest to members of the Association of Scientific Hydrology.

The Union's General Secretary, Colonal Laclavère, has been aware of our difficulties and has agreed to our proposal to publish independently in 1957 along the following lines.

In accordance with a decision made in Rome, this Bulletin will consist of two parts, one administrative and the other reserved for scientific publications. For financial reasons, we may be compelled at times to limit the scientific part, but we shall avoid this if possible. It is unnecessary to reiterate the reasons for the administrative part. This would be justified only in keeping you informed of the activities of other Associations and Committees, which are not members of our Union. The organisation of our Meetings and Symposia will be made easier by these communications.

2) The Secretary proposes to present in each number short communications, notices, etc., under the general heading of « Remarks ». I would ask you to pay particular attention to these if you wish to be kept informed of the Association's affairs. In the course of these remarks I shall take the liberty of adressing you informally.

3) I would ask you to make note of the following points. The present policy of the Union is to extend the organisation of Symposia by the Association, as for example the « Darcy » Symposia at Dijon, outside the General Assemblies. These Symposia draw attention to our work in those countries where our Assemblies are scarcely known, due to their infrequency.

UNESCO favours the organisation of these Symposia, but the Association's Treasurer is concerned about the cost of publication of these proceedings. The assistance of UNESCO is directed to the publication of these reports. The organisation and publications of the Dijon Symposia will cost about 6,000 dollars. At Dijon, the Council agreed to the proposal that the Transactions should not be distributed without payment. In spite of the efforts of some members, about whom I would like to say more later, the Dijon meeting will probably result in a financial loss. On the other hand, letters which I have received indicate that it was a most successful meeting from a scientific point of view.

4) I have just referred to those who have assisted in the distribution of the reports of the Dijon Symposia. I cannot mention individuals by name, but the thanks of the Association are due to those members who have made known these publications, particularly in their national journals.

In both Rome and Dijon, it was repeatedly said that our work is insufficiently well known. I ask you all again for your help. I believe that

short articles in national scientific journals have a favourable effect on our sales.

I apologise for these mundane considerations. I do not wish to raise commercial considerations, but our Association can only thrive if our feet are planted firmly on the ground.

TORONTO ASSEMBLY

1) April 1st was fixed as the last date for the return of titles of papers. National representatives were advised of this personally. I drew attention to this in the December 1956 Bulletin, but at the time of drafting this notice the total number of papers is not available. It would appear that certain countries have overlooked this date, and I shall accept titles until 30th April.

2) National representatives are asked to let me have their reports not later than 1st June, 1957, an I would ask that authors should not forget to include with their Reports a summary of a single page, typewritten and double spaced. These summaries should be forwarded by national representatives before the reports, if possible.

3) As a guide, a first draft of the programme of the various Commissions is given, subject to alteration.

Tuesday 3rd September
Morning: General Meeting of the Union. Convocation Hall.
Afternoon: Council Meeting of the Association.

Wednesday 4th September
Morning: Ordinary Meeting with the Oceanographic and Meteorological Associations. General theme: Hydrological balance.
The Hydrological Association will present some paper on this subject and probably on the influence of vegetation.
Afternoon: Ditto.

Thursday 5th September
Morning: Snow and Ice: 1st session.
Ground Water: 1st session.
Afternoon: General Meeting. Presidential address: Secretary's Report Outline of various questions and discussions.

Friday 6th September
Morning: Snow and Ice: 2nd session.
Ground Water: 2nd session.
Afternoon: Snow and Ice: 3rd session.
Ground Water: 3rd session.

Saturday 7th September
Morning: Snow and Ice: 4th session.
Ground Water: 4th session.

Sunday 8th September
Excursion.

Monday 9th September
Morning: Snow and Ice: 5th session.
Ground Water: 5th session.
Afternoon: Continental Erosion: 1st session.
Measures and Instruments.

Tuesday 10th September
Morning: Continental Erosion: 2nd session.
Rainfall (1st session).
Afternoon: Continental Erosion: 3rd session.
Surface Water: 1st session (Evaporation).

Wednesday 11th September
Morning: Surface Water: 2nd session (Evaporation).
Snow and Ice: 6th session.
Afternoon: Surface Water: 3rd session.
Snow and Ice: 7th session.

Thursday 12th September
Morning: Dew, condensation and similar precipitations: 1st session.
Afternoon: Ditto: 2nd session.
Rainfal (2nd session).

Friday 13th September
Morning: 9 a.m. Surface Water: 4th session.
11 a.m. Discussion on standardisation of hydrological characteristics.
Afternoon: 2.30 p.m. Surface Water: 5th session.
5 p.m. General Meeting.

Saterday 14th September
Morning: Closing Meeting of the Union.

UNESCO/NS/140
PARIS, le 17 janvier 1957
Traduit de l'anglais

ORGANISATION DES NATIONS UNIES POUR L'ÉDUCATION, LA SCIENCE ET LA CULTURE

PROGRAMME DE LA ZONE ARIDE

RAPPORT SUR LA ONZIÈME SESSION DU COMITE CONSULTATIF DE RECHERCHES SUR LA ZONE ARIDE

University College (Canberra)
13-16 octobre 1956
et
Melbourne
25 octobre 1956

1. Le Comité consultatif de recherches sur la zone aride a tenu sa onzième session du 13 au 16 octobre 1956 à l'University College de Canberra, et le 28 octobre à Melbourne.

2. L'Organisation des Nations Unies, les Institutions spécialisées et les organisations internationales non gouvernementales qui s'intéressent aux recherches scientifiques et techniques sur la zone aride avaient été invitées à envoyer des représentants. Le Directeur général a également invité les membres australiens des collèges de consultants spécialisés dans les recherches sur la zone aride à participer à la session, qui était publique. La liste des participants est donnée en annexe au présent rapport.

3. La session a été ouverte par M. G. Aubert (France), Président de la dixième session, qui a remercié le Gouvernement australien d'avoir invité l'Unesco à tenir cette session à Canberra, et le Président et le Conseil de l'University College d'avoir mis leurs locaux à la disposition du Comité.

4. Le Comité a porté M. H. O'R. Sternberg (Brésil) à la présidence pour la durée de la onzième session. Le Président a constitué un sous-comité (composé de MM. G. Aubert (France), M. H. Ganji (Iran) et Herbert Greene (Royaume-Uni) qu'il a chargé de rédiger le rapport sur la session.

5. Le Comité a souhaité la bienvenue à M. C. W. Thornthwaite (Etats-Unis d'Amérique) qui lui a été adjoint depuis la dernière session, ainsi qu'à M. M. H. Ganji, suppléant de M. A. Behnia (Iran), empêché d'assister à la session. Il a regretté que le troisième de ses nouveaux membres, M. G.V. Bogomolov (Union des Républiques soviétiques socialistes), venant de Moscou, ait été retardé en cours de route. Le Comité a d'autant plus déploré l'absence d'un représentant de l'Organisation des Nations Unies que le Conseil économique et social a, au cours de sa vingt et unième session (avril-mai 1956), adopté des résolutions chargeant le Secrétaire général d'entreprendre une action dans certains domaines auxquels le Comité porte un vif intérêt.

6. Le Comité a pris note avec une grande satisfaction des dispositions que le Directeur général et le Gouvernement australien ont adoptées en vue d'organiser à Canberra, du 17 au 20 octobre 1956, un colloque sur la climatologie de la zone aride, notamment sur la micro-climatologie, et des mesures que le Gouvernement australien a décidées pour permettre aux participants à ce colloque de faire ensuite un voyage d'étude. Le Comité a prié le Directeur

général d'exprimer sa gratitude au Gouvernement australien, à la Commonwealth Scientific and Industrial Research Organization (que le Gouvernement avait chargé de préparer le colloque), à M. B. T. Dickson, Président, et aux membres du comité local d'organisation.

7. Le Comité a approuvé le rapport du Secrétariat, qui rend compte de l'œuvre accomplie dans le cadre du programme de la zone aride depuis la session précédente. Il s'est déclaré satisfait de l'action exercée par le Secrétariat au cours de cette période. Il a remercié le représentant de l'Organisation météorologique mondiale qui lui a transmis les vœux de son Président, en réaffirmant l'intérêt que l'OMM porte aux travaux du Comité et le désir qu'elle a d'y prendre part. Il a également remercié de ses vœux le Conseil scientifique pour l'Afrique au sud du Sahara et a pris note avec satisfaction de l'activité exercée par ce Conseil dans le domaine des recherches sur la zone aride. Il a exprimé l'espoir que le Directeur général invitera désormais ce Conseil à se faire représenter aux sessions du Comité.

Projet majeur

8. Le Comité consultatif a étudié avec soin et dans le détail les propositions concernant un projet majeur relatif aux recherches scientifiques sur les terres arides, que le Directeur général a présentées dans le Projet de programme et de budget de l'Unesco pour 1957-1958, dont sera saisie la Conférence générale de l'Unesco lors de sa neuvième session, à New Delhi, en novembre 1956. Le Comité a formulé les recommandations suivantes :

Généralités

Recommandation 1

9. Il ressort nettement des débats de la Conférence des Nations Unies sur la conservation et l'utilisation des ressources naturelles (1948) que les problèmes des régions arides et semi-arides ont été négligés. C'est pourquoi l'Unesco a accepté la mission d'orienter l'attention des milieux scientifiques vers ces questions.

10. Les zones arides et semi-arides s'étendent sur de vastes territoires. Les conditions physiques y grèvent lourdement la vie végétale et animale. Les groupes humains, qui luttent pour survivre en milieu hostile, y sont faibles et isolés. Ils n'ont probablement pas assez — ou pas du tout — d'hommes de sciences et pourtant il n'est pas possible de résoudre leurs problèmes — ni même de les rendre moins aigus — autrement que par un effort continu faisant appel à la collaboration de plusieurs sciences. A ce prix, on devrait pouvoir accomplir des progrès sur un vaste front.

11. Le comité consultatif de recherches sur la zone aride, considérant la modicité des ressources dont dispose l'Unesco, a rejeté les propositions tendant à les consacrer à la création d'un institut ou d'un laboratoire international de recherches sur la zone aride, qui risquerait d'en absorber l'intégralité. Le Comité consultatif a décidé que ce qu'il peut faire de plus utile est de :

a) Rassembler et diffuser des informations en dressant l'inventaire des recherches et en organisant des colloques;

b) Faciliter la planification, la coordination et l'exécution des recherches sur la zone aride par l'intermédiaire d'institutions reconnues et grâce au concours de personnes spécialement choisies;

c) Accorder des bourses pour la formation de spécialistes en matière de recherches sur la zone aride.

12. Une importante addition au programme a été approuvée à l'issue d'une réunion tenue à Socorro (avril-mai 1955) conjointement avec l'American Association for the Advancement of Science. Il a été observé que, même dans les pays où la valeur des ressources scientifiques est amplement reconnue, on néglige incontestablement d'appliquer aux problèmes des terres arides les connaissances acquises. Ce fait est imputé non seulement au manque de coordination entre les diverses sciences, mais aussi au manque de contact entre les savants et les profanes. Il a été décidé qu'il y avait lieu de s'employer davantage à créer des comités locaux et nationaux comprenant de nombreuses personnalités non scientifiques.

13. Le Comité consultatif est convaincu qu'un grand pas sera fait dans cette direction si les Postes régionaux de coopération scientifique qui doivent faciliter la réalisation du projet majeur, considèrent qu'ils ont pour mission primordiale d'encourager et d'appuyer l'action de ces comités locaux et nationaux. Des crédits additionnels seront nécessaires à cet effet.

14. Les membres des Postes de coopération scientifique, en nouant des relations cordiales avec les membres des comités locaux et nationaux, doivent provoquer des demandes raisonnables d'assistance en vue de projets de recherches s'inscrivant dans le cadre du programme relatif à la zone aride. Des projets bien conçus, émanant des comités locaux ou nationaux, et aboutissant à des résultats tangibles, peuvent très efficacement contribuer à éliminer les obstacles aux progrès. Une fois bien comprise des milieux locaux, la tâche entreprise par l'Unesco pourrait être puissamment soutenue par l'une ou l'autre des très riches collectivités installées en zone aride.

15. Les conditions ainsi créées faciliteraient l'établissement d'un programme judicieusement coordonné dans lequel les comités locaux et nationaux d'une part, et l'Unesco, d'autre part, se répartiraient la tâche.

Recommandation 2

16. Le Comité, considérant que les limites des régions arides et semi-arides sont dynamiques plutôt que statiques, que de nouveaux Etats pourront devenir membres de l'Unesco au cours des cinq ou six prochaines années, et que, dans la liste incorporée au préambule du projet majeur plusieurs territoires non autonomes ne sont pas mentionnés, a recommandé d'éliminer du texte du projet majeur la liste nominative des pays.

A. — Sessions du Comité consultatif

Recommandation 3

17. Considérant qu'il importe d'entreprendre aussitôt que possible en 1957, la tâche prévue en faveur du Moyen-Orient dans le cadre du projet majeur, et tenant compte de la proposition tendant à ce que l'exécution de ce projet se poursuive pendant cinq ou six ans, le Comité a recommandé que le Directeur général envisage à titre exceptionnel de convoquer deux sessions du Comité en 1957.

B — Comités nationaux et locaux de coopération

Recommandation 4

18. Le Comité a recommandé que des mesures soient prises pour faire en sorte que des exemplaires du Guide des travaux de recherche sur la mise

en valeur des régions arides (Guidebook to research data for arid zone development) soient mis à la disposition des comités nationaux et locaux de coopération, des instituts de recherches sur la zone aride, ainsi que d'organisations et de personnalités spécialement choisies qui pourraient assurer l'utilisation rationnelle de cet ouvrage.

Recommandation 5

19. Le Comité a recommandé que le Secrétariat publie un bulletin périodique ayant pour principal objet d'établir un lien entre les Comités de coopération et le Comité consultatif. Ce bulletin contiendrait des renseignements sur les activités entreprises dans le cadre du programme, sur les réunions scientifiques dont les travaux touchent à la zone aride, notamment celles auxquelles participent des membres du Comité consultatif, et donnerait des informations sur les comités de coopération. Il fournirait aussi des indications sur les bourses. Le premier numéro contiendrait un article exposant la façon dont se constitue un comité national ou local de coopération. Le montant nécessaire à la publication de ce bulletin pourrait être prélevé sur les crédits inscrits au budget aux fins de publication d'une deuxième édition du *Directory of Institutions engaged in Arid Zone Research* (Répertoire des institutions s'occupant de recherches sur la zone aride) car, de l'avis du Comité, ce répertoire serait peu demandé, à en juger par le faible chiffre de vente de la première édition.

C — Rapports-inventaires sur les recherches

Recommandation 6

20. Le Comité a recommandé qu'en 1957 un contrat soit passé pour la rédaction d'un ouvrage retraçant l'historique du mode d'utilisation des terres et notamment de l'agriculture, dans les régions arides et semi-arides : cet ouvrage prendrait place dans la collection des rapports-inventaires non périodiques. Le Comité attache une grande importance à ce travail.

Recommandation 7

21. Le Comité a recommandé qu'un contrat soit passé en 1957 pour rédaction, aux fins de publication dans la collection des inventaires périodiques, de deux publications dont la première serait un inventaire critique sélectif des principales contributions apportées, depuis le Colloque d'Ankara de 1952, au domaine de l'hydrologie, du point de vue particulier de la zone aride, et la seconde un inventaire analogue des travaux effectués au sujet des plantes médicinales de la zone aride. Le Comité a pris note avec satisfaction de l'offre qu'a faite l'Organisation mondiale de la santé d'aider le Comité à établir ce dernier inventaire. Ces deux inventaires, qui ne comprendraient qu'une quarantaine de pages imprimées, seraient publiés séparément.

D — Colloques

Recommandation 8

22. Le Comité a pris note avec un vif intérêt du projet d'organisation, au cours du troisième trimestre 1957, d'un colloque sur la zone aride sous les auspices du Gouvernement du Pakistan. Il a recommandé que l'Unesco, au cas où elle y serait conviée, accorde aux organisateurs du dit colloque

une assistance financière leur permettant d'inviter des savants étrangers à participer aux débats. Le Comité consultatif pourrait fort bien faire coïncider avec ce colloque sa deuxième session de 1957.

Recommandation 9

23. Le Comité a recommandé que l'Unesco s'entende avec un Etat membre en vue d'organiser un colloque en 1958, Il a proposé que ce colloque soit consacré à l'étude de la salinité des sols et des eaux, et notamment à la purification des eaux salines, ainsi qu'à l'utilisation des eaux salines par les plantes, les animaux et l'homme.

H — **Projets spéciaux**

Recommandation 10

24. Le Comité a pris note avec un grand intérêt du vœu émis par le groupe de travail n° 8 (purification des eaux salines) de l'Organisation européenne de coopération économique dont l'objectif est maintenant atteint, tendant à ce qu'une organisation se charge d'assurer la liaison entre les membres du groupe et les autres intéressés, de diffuser les renseignements qui offrent de l'intérêt pour le groupe, et d'organiser de temps à autre des échanges de vues sur les problèmes abordés et les résultats obtenus. Le Comité a estimé que le programme de la zone aride permet de donner satisfaction à ce vœu, notamment par le moyen du bulletin, des rapports — inventaires et des colloques. Il a recommandé en conséquence que le Directeur général accueille favorablement la demande du Secrétaire général de l'OECE. Il a, en outre, recommandé qu'un colloque sur les problèmes relatifs à la salinité soit organisé dans les plus brefs délais (voir recommandation 8). Ce colloque aurait pour objet d'adopter des recommandations touchant les travaux que le Comité devra entreprendre dans ce domaine. Une étroite coopération devrait être assurée avec le « Saline Water Conversion Programme » réalisé aux Etats-Unis d'Amérique.

I — **Publications**

Recommandation 11

25. Considérant que le projet majeur intéressera au premier chef les pays du Moyen-Orient, considérant d'autre part l'intérêt qu'il y a à porter, dans toute la mesure du possible, les renseignements concernant la zone aride à la connaissance de tous ceux qui pourraient en profiter, le Comité a recommandé que des mesures soient adoptées en vue de favoriser la publication dans les langues de la région des principaux résultats obtenus.

RÉUNIONS SCIENTIFIQUES

Recommandation 12

26. Le Comité a entendu avec un vif intérêt les exposés de ceux de ses membres qui ont participé au Congrès international de géographie, au Congrès international de pédologie et au Stage d'études international sur la géographie réuni par l'Université musulmane d'Aligarh (Inde). Il a recommandé que ses membres rédigent à l'intention du Secrétariat de brefs rapports sur ces réu-

nions, aux fins de reproduction et de diffusion. Il serait bon que les membres du Comité prennent désormais l'habitude d'établir ainsi, dans la mesure du possible, sur les réunions scientifiques auxquelles ils participent, des rapports présentés selon un modèle uniforme.

PRÉPARATION D'UNE CARTE AVEC LE CONCOURS DU COMITÉ DE RECHERCHES SUR LA ZONE TROPICALE HUMIDE

Recommandation 13

27. Le Comité a pris acte de la recommandation faite par la Réunion préliminaire de spécialistes des recherches sur la zone tropicale humide, tenue à Kandy (Ceylan), en mars 1955, touchant l'organisation d'une réunion mixte de représentants des Comités de recherches sur la zone aride et sur la zone tropicale humide en vue de préparer une carte à échelle réduite délimitant dans leurs grandes lignes les régions du monde où se posent des problèmes intéressant les deux programmes. Il a également entendu un rapport de M. G. Aubert et de M. S. N. Naqvi (participants à la réunion de Kandy) sur les raisons pour lesquelles cette recommandation a été adoptée. Le Comité a invité le Directeur général à signaler à l'attention du Comité consultatif de recherches sur la zone tropicale humide les cartes homoclimatiques publiées en 1951 dans le cadre du programme de la zone aride. Ces cartes délimitent les zones qui offrent un intérêt particulier pour les recherches sur la zone aride. Au cas où le Comité consultatif de recherches sur la zone tropicale humide jugerait opportun de dresser des cartes spécialement conçues à son intention et souhaiterait obtenir la coopération du Comité de recherches sur la zone aride, celui-ci étudiera bien volontiers avec lui les problèmes qui offrent un intérêt commun.

ÉNERGIE SOLAIRE

Recommandation 14

28. Le Comité a pris acte avec intérêt :

a) de la recommandation par laquelle le Comité consultatif international de la recherche dans le programme des sciences exactes et naturelles invite l'Unesco à maintenir la contribution qu'elle apporte au perfectionnement et à l'uniformisation des instruments de mesure de la radiation solaire; et

b) de la création par l'Association internationale de météorologie d'une sous-commission chargée de coordonner les recherches relatives à la radiadiation solaire et aux utilisations pratiques de l'énergie solaire.

Il a exprimé le désir d'être tenu au courant de toutes les activités entreprises dans ce domaine tant par le Directeur général que par la dite sous-commission.

RAPPORTS DÉFINITIFS SUR LES PROJETS DE RECHERCHES BÉNÉFICIANT D'UNE ASSISTANCE

Recommandation 15

29. Le Comité a pris acte de la réception des rapports définitifs concernant les projets suivants :

a) Recherches sur la rosée, par le Professeur B. Hellstrom.

b) Recherches sur la rosée, par Mlle I. Arvidsson.

c) Etude phytosociologique de la flore du désert du Rajasthan, par le Professeur F. R. Bharucha.

d) Recherches écologiques et éthologiques sur le chameau dans le nord-ouest du Sahara, par Mme G. Hautier-Pilters.

En ce qui concerne le rapport d), le Comité estime qu'il s'agit là d'un travail de grande valeur. Il invite le Secrétariat à s'informer s'il serait utile d'apporter une contribution financière à la publication des conclusions ou à l'exécution des recherches complémentaires mentionnées à la fin du dit rapport.

RAPPORTS INTÉRIMAIRES

Recommandation 16

30. Le Comité a pris connaissance avec grand intérêt du rapport de M. P. Meigs sur l'étude des possibilités géographiques des déserts côtiers. Il aimerait recevoir le rapport définitif concernant ce projet avant sa prochaine session.

31. Le Comité a pris connaissance avec un vif intérêt du rapport du Professeur O. Stocker sur les recherches écologiques qu'il a effectuées en Mauritanie.

32. Le Comité a également reçu le rapport sur l'étude de la flore du Rajpoutana établie par le Professeur S. Sarup.

33. Le Comité a écouté avec un grand intérêt le compte rendu du voyage fait par le Professeur Emberger au Moyen-Orient au printemps de 1956. Il sera également heureux de prendre connaissance du rapport écrit sur le voyage, que le Secrétariat a été prié de reproduire et de distribuer.

DEMANDES D'ASSISTANCE

Recommandation 17

34. Le Comité a examiné la requête du Professeur Franz, de l'Université de Vienne. Il ne s'est pas trouvé en mesure de recommander qu'une subvention soit accordée sous la forme requise au titre du voyage en Afrique Equatoriale française projeté par le Professeur Franz : aucune assurance n'est en effet fournie en ce qui concerne la nature de l'expédition en question, ses objectifs et les facilités qu'elle offrira pour la conduite de travaux scientifiques. Le Comité a accepté avec gratitude l'offre de M. Aubert qui s'est déclaré prêt à aider le Professeur Franz à organiser ses travaux sur le terrain. Il a recommandé qu'une aide soit accordée au Professeur Franz quand le Secrétariat aura reçu de M. Aubert les plans complets du projet.

Recommandation 18

35. Sur la base de la recommandation 13 f) faite lors de sa 10me session, le Comité a recommandé que le Laboratoire de climatologie de Centerton, N.J. (Etats-Unis d'Amérique) soit chargé de mener à bien l'établissement de deux séries de cartes climatiques du Moyen-Orient, chaque série devant comprendre deux feuilles. Ces cartes indiqueraient la pluviosité et l'évaportanspiration potentielle. Une fois achevées, elles seraient publiées. si possible par l'Unesco, en même temps qu'une brochure expliquant quels principes ont présidé à leur élaboration et comment on doit les utiliser. La dépense envisagée par le Comité au titre de ce projet s'élève au total à 10.000 dollars.

Recommandation 19

36. Le Comité a pris acte avec un grand intérêt du rapport présenté par l'Institut français d'Afrique noire sur les travaux scientifiques effectués pendant l'année écoulée en ce qui concerne les enclos aménagés à Adar (Afrique Occidentale française) en vue de l'étude de la dynamique des groupements végétaux. Il n'a pas été en mesure de recommander qu'une aide soit accordée en vue de l'achat de l'équipement scientifique nécessaire à cette étude, son règlement interdisant l'emploi des fonds de l'Unesco pour l'achat d'équipement permanent, mais il a recommandé que le Directeur général alloue l'équivalent de 700 dollars à l'Institut pour l'aider à entretenir et à protéger les enclos.

ACTIVITÉS PROPOSÉES PAR LE POSTE DE COOPÉRATION SCIENTIFIQUE DU MOYEN-ORIENT POUR 1957

Recommandation 20

37. Le Comité a examiné les plans établis par le PCSMO en vue de participer au Troisième Congrès scientifique arabe qui se tiendra sans doute à Damas en août 1957, et de fournir une assistance au Colloque sur l'entomologie du Moyen-Orient qui doit se réunir à l'occasion du cinquantième anniversaire de la Société égyptienne d'entomologie.

38. Il apparaît que l'organisation d'une réunion consacrée spécialement aux problèmes de la zone aride, ainsi que celle d'une exposition présentée dans le cadre du Congrès, seraient extrêmement utiles. Le Comité pourrait être représentée à cette réunion par le Professeur Thacker, qui doit se trouver au Moyen-Orient à l'époque indiquée, et peut-être par un autre de ses membres. Cette question sera examinée de nouveau au cours de la prochaine session; à ce moment il se peut que le Poste de coopération scientifique du Moyen-Orient soit à même de communiquer des renseignements plus détaillés sur le Congrès et sur l'organisation du Colloque entomologique du Caire.

ASSISTANCE TECHNIQUE

Recommandation 21

39. Le Comité a pris connaissance avec un grand intérêt du rapport final de la mission d'assistance technique chargée de s'occuper de la création d'un Institut hydrogéologique en Turquie. Il a demandé au Secrétariat de lui fournir, dans la mesure du possible, pour examen, des exemplaires de tous les rapports de ce genre.

COOPÉRATION AVEC LA FAO EN MATIÈRE DE LUTTE CONTRE LES ACRIDIENS DU DÉSERT

Recommandation 22

40. Le Comité a pris connaissance avec intérêt et satisfaction de la recommandation faite à Londres, en avril 1956, par le groupe d'experts de la FAO sur les mesures à long terme de lutte contre les acridiens du désert; ces experts ont recommandé qu'une étude écologique générale des principales aires de reproduction des acridiens du désert soit menée, et que la FAO invite le

Comité consultatif à y participer. Le Comité aimerait recevoir de la FAO des indications plus concrètes et plus détaillées sur le concours qu'il pourrait fournir, afin de les examiner lors de sa prochaine session. Il estime qu'il y aurait grand intérêt à rattacher au projet majeur une entreprise collective de ce genre, à laquelle l'OMS pourrait elle aussi coopérer.

CONCLUSIONS GÉNÉRALES DU COLLOQUE DE CANBERRA SUR LA CLIMATOLOGIE ET PLUS SPÉCIALEMENT LA MICRO-CLIMATOLOGIE DE LA ZONE ARIDE

Recommandation 23

41. Le Comité a constaté avec satisfaction que les discussions qui ont eu lieu au cours du Colloque ont mis en lumière la nécessité d'entreprendre des recherches sur différents problèmes concernant la zone aride, tels que les suivants :

Rapports entre la composition des collectivités végétales et le régime hydrique et thermique; présence et utilisation des eaux souterraines, de la rosée et de la vapeur d'eau; maxima et minima de température et de luminosité qui influent sur la germination des graines; croissance et survivance des plantes dans les régions arides; influence exercée à cet égard par la nutrition et par la nature du sol; possibilité d'utiliser des données recueillies par un vaste réseau d'observateurs en ce qui concerne l'application d'indices climatiques; étude des populations d'insectes, d'oiseaux et d'autres animaux de la zone aride et de la physiologie de ces espèces; rôle et propriétés de l'isolement et de la pigmentation des couches superficielles; « proprioclimat » des individus; établissement des plans de laboratoires d'essai fixes et mobiles; distribution et effets biologiques des rayons ultra-violets.

42. Comme l'étude de l'évaporation dépend essentiellement de la mesure de la perte d'eau, il est évident que pour faciliter les recherches concernant les divers problèmes ci-dessus, il faudrait que cette mesure soit régulièrement assurée dans le plus grand nombre possible de stations climatologiques; il faudrait aussi mesurer la température superficielle des lacs, des étangs et des cuvettes, ainsi que la vitesse du vent à une altitude suffisante pour que l'influence de la nature du terrain ne se fasse plus sentir. Il serait préférable que les observations soient résumées sous la forme de totaux portant sur cinq ou dix jours; en ce qui concerne les radiations, la création de réseaux de postes d'observation utilisant des instruments robustes et d'un prix relativement modique est extrêmement souhaitable.

ANNEXE

LISTE DES PARTICIPANTS

Membres du Comité consultatif

M. G. Aubert (France)
le Professeur G. V. Bogomolov (Union des Républiques soviétiques socialistes)
M. B. T. Difcson (Australie)
M. M. H. Ganji (suppléant du Professeur A. Behnia (Iran))
M. H. Greene (Royaume-Uni)
M. S. N. Naqvi (Pakistan)
le Professeur R. E. G. Pichi-Sermolli (Italie)

le Professeur H. O'Reilly Sternberg (Brésil)
le Professeur M. S. Thacker (Inde)
M. C. W. Thornthwaite (Etats-Unis d'Amérique)

Représentants des Institutions spécialisées des Nations Unies

Organisation des Nations Unies pour l'alimentation et l'agriculture : M. R. O. Whyte (Rome) et M. H. C. Forester (CSIRO, Australie)
Organisation mondiale de la santé : M. R. N. Clark (Genève)
Organisation météorologique mondiale : M. H. T. Ashton et M. A. Gibbs (Bureau of Meteorology, Brisbane)

Représentants d'organisations internationales non gouvernementales scientifiques et techniques

Conseil international des unions scientifiques : le Professeur E. S. Hills, de l'Université de Melbourne
Union des associations techniques internationales : le Professeur C. W. N. Sexton, de l'Université de Melbourne.
Union géographique internationale : le Professeur H. O'Reilly Sternberg
Association internationale d'hydrologie : le Professeur Sexton
Conseil scientifique pour l'Afrique au Sud du Sahara : M. G. Aubert.

Expédition Glaciologique Internationale au Groenland

RÉUNION DE PARIS (14-16 décembre 1956)

Nous ne sommes pas encore en possession du compte rendu officiel de cette réunion.

Toutefois, nous considérons devoir présenter dès maintenant le texte suivant présenté à cette réunion par le Secrétaire de l'Association, texte qui précise la nature des relations existant entre l'AIHS et l'EGIG.

Patronage de l'A. I. H. S.

1) En accordant son patronage à l'E. G. I. G., l'A. I. H. S. marque tout l'intérêt qu'elle porte à l'entreprise considérée.

L'A. I. H. S. et sa Commission des Neiges et des Glaces étant les hautes autorités scientifiques internationales dans le domaine de la glaciologie, l'E. G. I. G. peut profiter de ce patronage, c'est-à-dire de la marque d'intérêt qui lui est manifestée pour s'en prévaloir auprès des organismes capables de soutenir financièrement ou moralement l'entreprise ou de l'aider de façon quelconque.

2) Il est nettement entendu que l'A. I. H. S. n'interviendra pas financièrement dans l'entreprise.

3) Toutefois, comme marque spéciale de l'intérêt qu'elle porte à ces travaux, l'A. I. H. S. fera paraître dans ses comptes rendus ou son bulletin, les résultats spécifiquement hydro-glaciologiques de l'expédition.

4) L'A. I. H. S. en se portant en fait garante de la valeur scientifique du *but* poursuivi par l'E. G. I. G. et les membres de son comité directeur, peut en retour exiger que ces membres soient agréés par elle et par suite par les Comités nationaux qui, d'après les statuts de l'Union et de l'Association, choisissent les membres qui représentent chacun des pays participants.

5) L'E. G. I. G. peut devenir une sous-commission de la Commission des Neiges et des Glaces de l'A. I. H. S. par un vote de l'Assemblée Générale.

AVIS DE SOCIÉTÉS

SOCIÉTÉ HYDROTECHNIQUE DE FRANCE

199, rue de Grenelle, Paris (VII^{e})
Invalides 13-37

V^{es} JOURNEES DE L'HYDRAULIQUE
AIX-EN-PROVENCE 23/28 JUIN 1958

La « Société Hydrotechnique de France » organise les *Cinquièmes Journées de l'Hydraulique — l'année prochaine* — du 23 au 28 juin 1958, sur le thème général :

Turbines et Pompes hydrauliques

qui comportera :
— un exposé introductif sur les divers types de turbines et de pompes hydrauliques, et sera subdivisé en 7 questions :

I — Evolution de la technologie des turbines et son influence sur le Génie Civil
II — Régulateurs des turbines
III — Rendement des turbines et problèmes de cavitation
IV — Exploitation des turbines
V — Turbines - Pompes
VI — Pompes
VII — Turbines de forage.

Les séances de travail auront lieu à Aix-en-Provence les 26, 27 et 28 juin 1958 et seront *précédées* d'un *voyage d'études* facultatif permettant de visiter des ateliers français de Construction de Turbines Hydrauliques et des chantiers et aménagements hydroélectriques.

Les personnes désireuses *de présenter des rapports* ou de prendre part aux « V^{mes} Journées de l'Hydraulique » sont priées *de le faire connaître dès maintenant* à la Société Hydrotechnique de France.

Un résumé en dix lignes de chaque mémoire devra être adressé en 3 exemplaires à la Société Hydrotechnique de France *avant le* 31 *octobre* 1957. *Les rapports* retenus devront être remis in extenso *au plus tard le* 28 *février* 1958.

Les rapports seront présentés, discutés et, éventuellement, publiés en *français*.

(Preliminary Program)

FIRST UNITED STATES INTERSOCIETY CONFERENCE
on
IRRIGATION AND DRAINAGE

Conference Theme **Can man develop a permanent irrigation agriculture**?
April 29 and 30, 1957 - Sheraton-Palace Hotel, San Francisco, Calif., U. S. A.

Sponsored by :

American Society of Civil Engineers (Irrigation and Drainage Division)

American Society of Agricultural Engineers (Soil and Water Division)
Soil Science Society of America (Conservation, Irrigation, Drainage and Tillage Division)

In cooperation with

U. S. National Committee of the International Commission on Irrigation and Drainage

CAN MAN DEVELOP A PERMANENT IRRIGATION AGRICULTURE?

Monday, April 29, 9 :30 AM

Purpose of Conference.
Teamwork in the Solution of Water Problems.

Water in a permanent Irrigation agriculture

Influence of Climate on Irrigation Agriculture.
Watershed Management in Relation to Water Yield

I. Water Yield Control Through Forest Management in Snow Pack Watersheds.
II. Water Yields as Influenced by Vegetation Management on Watersheds below Snowline.
III. Importance of Phreatophytes in Water Supply.

Monday, April 29, 2 :00 PM

Water in a permanent irrigation agriculture (Continued)

Factors Affecting the Useful Life of Reservoirs.
Continued Utilization of Ground Water Storage Basins.
Can Evaporation Losses be Reduced?
Problems of Maintaining a Satisfactory Water Quality.

Crops in a permanent irrigated Agriculture

Selection of Crops for Water Deficient Areas
Cropping Practices for Maximizing Water Use Efficiency.
Plant Diseases, Insects, and Weeds as Affected by Irrigation.

Tuesday, April 30, 9 :30 AM

Soils in a permanent irrigation agriculture

Soil Classification in Relation to the Irrigability of Land Areas
Effect of Irrigation on Soil Fertility and Profile Development.
Maintenance of Favourable Soil Structure and Adequate Water Infiltration Rates.
Salinity and Alkalinity Hazards.
Hydraulics of Irrigation Water Application as Affected by Soil and Other Factros.
Drainage in Relation to a Permanent Irrigation Agriculture.

Tuesday, April 30, – :45 PM

Man in permanent irrigation agriculture

Conflicting Demands for Water—Some Approaches to the Problem.
Irrigation Organizations and Water Law As Factors in a Permanent Agriculture.
Interstate and International Water Compacts.
Man's Changing Food Habits and Their Impact on Irrigation Agriculture.
Health Problems as Affected by Irrigation Agriculture.
Man's Opportunities to Assure the Permanence of Irrigation Agriculture—A Summary.

The programme is being organized by Mr. Robert M. Hagan, Chairman, Department of Irrigation, University of California, Davis, California. Member: ASAE, SSSA, ICID. The best available authorities in the U.S.A. in each field are being requested to participate in the programme.

INTERNATIONAL COMMISSION ON IRRIGATION AND DRAINAGE
THIRD CONGRESS
San Francisco, April 29 - May 4, 1957

Under the sponsoership of the United States National Committee

Program

Monday, April 29: Registration
Tuesday, April 30: Registration
Wednesday, May 1: 10:30 a.m. Inaugural session; 2:00 p.m Technical session, Question No. 8.
Thursday, May 2: 9:30 a.m. Technical session, Question No. 7.
Friday, May 3: 9:30 a.m. Technical session, Question No. 9.
Saturday, May 4: 9:30 a.m. Technical session, Question No. 10.
Sunday, May 5: Local Visits
Monday, May 6 to Friday, May 17: Study tours.
Saturday, May 18: 11:00 a.m. Closing session at Los Angeles.

Registration, inaugural, and technical sessions from April 29 to May 4 will be held at the Sheraton-Palace Hotel, San Francisco. The closing session on May 18 will be held at Los Angeles.

The Eighth Meeting of the International Executive Council will be held at the Sheraton-Palace Hotel on Tuesday, April 30, at 10:00 a.m.

Local visits will be arranged both in San Francisco and in Los Angeles, and there will be a separate program for the ladies.

The Irrigation and Drainage Division of the American Society of Civil Engineers will hold a Conference on April 29 and 30, and a joint meeting with the Pacific Coast Section of the American Society of Agricultural Engineers and the Soil Scientists Association of America on April 30. Delegates to the Third Congress are invited to the Conference and the joint meeting.

Questions to be discussed at the Third Congress

As previously announced in Bulletin No. 1, the following four Questions will be discussed at the Third Congress:

Question No. 7—*Canal lining*

Scope of the subject—Purpose and practical aspects, particularly materials, technical and economic aspects, maintenance and related problems.

Question No. 8—*Soil water relationship in irrigation*

Scope of the subject—Movement of moisture in irrigated soils, special methods for the preservation of soil structure and fertility and the effects of soil fertility on duty of irrigation water. Economical use of irrigation water.

Question No. 9—*Hydraulic structures on irrigation and drainage systems*

Scope of the subject—Particularly those relating to regulation and safety, distribution and measurement of water, prevention of silting (exclusive of canals, conduits and river works).

Question No. 10—*Interrelation between irrigation and drainage*

Participants in the Congress

Delegates from participating countries or from other organizations invited by the Central Office of the International Commission on Irrigation and Drainage, accompanied by their families, are invited to take part in the Congress.

Engineers, scientists, and technical specialists from non-participating countries, accompanied by their families, may also take part in the Congress as individual participants, provided their applications are received by the central Office in New Delhi and by the U. S. National Committee of the Commission, not later than March 1, 1957. Applications from Western Hemisphere countries should be mailed to the U. S. National Committee.

INTERNATIONAL ASSOCIATION FOR HYDRAULIC RESEARCH
ASSOCIATION INTERNATIONALE DE RECHERCHES HYDRAULIQUES

SEPTIEME CONGRES - LISBONNE

Juillet 1957

Sujets à traiter

Le programme définitif des sujets à traiter a été fixé comme suit :

A — *Effet d'échelle* — Principes généraux, études fondamentales, applications aux travaux hydrauliques et aux machines.

B — *Cavitation* — Principes généraux, études fondamentales, applications aux travaux hydrauliques et aux machines.

C — *Hydraulique des travaux de prise d'eau, de tunnels et canaux d'amenée, de tunnels de dérivation provisoire.*

D — *Sujets libres* — En ce qui concerne les sujets libres nous voulons préciser qu'ils ne pourront être présentés qu'après approbation par le Conseil de l'Association, ceci afin d'éviter d'alourdir les travaux du Congrès et de recevoir des rapports dont les sujets s'écarteraient trop des thèmes choisis.

COURS DE RECHERCHES SCIENTIFIQUES EN HAUTE MONTAGNE

(Gletscherkurs) du Prof. R. Finsterwalder

Suivant une tradition bien établie, le Professeur Finsterwalder organise un cours de recherches scientifiques en haute montagne.

M. le Professeur Finsterwalder m'a communiqué, à ce sujet, l'avis suivant :

Kurs fur Hochgebirgsforschung 1957

Der Kurs findet vom 18 bis 25 August 1957 in der Alpinen Forschungsstelle der Universität Innsbruck in Obergurgl, Otztal, Tirol, statt. Er wird durch Herrn Prof. Dr. H. Kinzl, Innsbruck, und den Unterzeichneten geleitet.

Das Forschungs- und Unterrichtungsprogramm des Kurses umfasst :

Photogrametrische Aufnahme des derzeitigen Gletscherstandes in den Zungengebieten des Gurgler- und Rotmossferners sowie einzelne Aufnahmen in den Firngebieten. Gletschergeschwindigkeitsmessungen nach der photogrammetrischen Methode und mit dem Theodolit.

Morphologische, geologische und pflanzengeographische Forschungen in den Gletschervorfeldern, insbesondere in den durch den Gletscherrückgang seit kurzem eisfrei gewordenen Gebieten, dazu geographische Aufnahmen und Untersuchungen in den Moränengebieten. Studium und Aufnahme der blockgletscher. Nach Möglichkeit geophysikalische und meteorologische Forschungen.

Theoretische und praktische Einführung in die modernen Hilfsmittel der Gletscherforschung, insbesondere die photogrammetrische Aufnahme-methode, sowie Einführung in die topographische und kartographische Aufnahme bei Forschungsreisen.

Die Zahl der Teilnehmer ist beschränkt. Alpine Erfahrung und körperliche Eignung für die teilweise anstrengenden Arbeiten im Gelände sind Voraussetzung. Erwünscht sind vor allem Meldungen von solchen Teilnehmern, die sich weiterhin den Aufgaben der Gletscher- und Hochgebirgsforschung im In- und Ausland sowie der Polarforschung aktiv widmen wollen. Einige Vertrautheit mit geodätischen Messinstrumenten und ihren Gebrauch ist notwendig.

Die Kursgebühr beträgt DM 50,—; sie kann auf begründeten Antrag erlassen werden.

Le cours aura lieu du 18 au 25 août 1957 à Obergurgl-Oetztal, Autriche.

Pour tout renseignement s'adresser à Monsieur le Professeur R. Finsterwalder, Institut für Photogrammetrie, Arcisstrasse 21, München (Allemagne)

DARCY'S LAW AND THE FIELD EQUATIONS OF THE FLOW OF UNDERGROUND FLUIDS

BY

M. KING HUBBERT
Chief Consultant (General Geology)

Paper Commemorating the
Centennial of Darcy's Law

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DARCY'S LAW AND THE FIELD EQUATIONS OF THE FLOW OF UNDERGROUND FLUIDS

BY

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ABSTRACT

In 1856 Henry Darcy described in an appendix to his book, *Les Fontaines Publiques de la Ville de Dijon*, a series of experiments on the downward flow of water through filter sands, whereby it was established that the rate of flow is given by the equation:

$$q = -K(h_2 - h_1)/l\,,$$

in which q is the volume of water crossing unit area in unit time, l is the thickness of the sand, h_1 and h_2 the heights above a reference level of the water in manometers terminated above and below the sand, respectively, and K a factor of proportionality.

This relationship, appropriately, soon became known as Darcy's law. Subsequently many separate attempts have been made to give Darcy's empirical expression a more general physical formulation, with the result that so many mutually inconsistent expressions of what is purported to be Darcy's law have appeared in published literature that sight has often been lost of Darcy's own work and of its significance.

In the present paper, therefore, it shall be our purpose to reinform ourselves upon what Darcy himself did, and then to determine the meaning of his results when expressed explicitly in terms of the pertinent physical variables involved. This will be done first by the empirical method used by Darcy himself, and then by direct derivation from the Navier-Stokes equation of motion of viscous fluids. We find in this manner that:

$$\vec{q} = (Nd^2)(\rho/\mu)[\vec{g} - (1/\rho)\,\mathrm{grad}\,p] = \sigma\vec{E}\,,$$

is a physical expression for Darcy's law, which is valid for liquids generally and for gases at pressures higher than about 20 atmospheres. Here N is a shape factor and d a characteristic length of the pore structure of the solid, ρ and μ are the density and viscosity of the fluid, $\sigma = (Nd^2)(\rho/\mu)$ is the volume conductivity of the system, and $\vec{E} = [\vec{g} - (1/\rho)\,\mathrm{grad}\,p]$ is the impelling force per unit mass acting upon the fluid. It is found also that Darcy's law is valid only for flow velocities such that the inertial forces are negligible as compared with those arising from viscosity.

In general three-dimensional space there exist two superposed physical fields: a field of force of caracteristic vector $\vec{E}$, and a field of flow of vector $\vec{q}$. The force field is more general than the flow field since it has values in all space capable of being occupied by the fluid.

So long as the fluid density is constant or is a function of the pressure only,

$$\mathrm{curl}\,\vec{E} = 0\,,\ \vec{E} = -\,\mathrm{grad}\,\Phi$$

where

$$\Phi = gz + \int\frac{dp}{\rho}\,.$$

The field of flow, independently of the force field, must satisfy the conservation of mass, leading to the equation of continuity

$$\operatorname{div} \rho\vec{q} = -f\, \partial\rho/\partial t$$

where f is the porosity, and t is the time. For steady motion $\partial\rho/\partial t = 0$, and

$$\operatorname{div} \rho\vec{q} = 0\,.$$

If the fluid is also of constant density,

$$\operatorname{div} \vec{q} = 0\,.$$

The two fields are linked together by Darcy's law,

$$\vec{q} = \sigma\vec{E}\,,$$

which is physically analogous to Ohm's law in electricity.

Then, when $\vec{E} = -\operatorname{grad} \Phi$,

$$\vec{q} = -\sigma \operatorname{grad} \Phi\,.$$

By means of the foregoing equations the flow of both homogeneous and heterogeneous fluids through porous solids becomes amenable to the same kind of analytical treatment as is already familiar in electrical and thermal conduction.

The relation of Darcy's work to the development of a valid theory of the flow of fluids through porous solids, is somewhat analogous to that of Faraday to the Maxwellian equations of electromagnetism. It forms a solid experimental foundation for such a field theory, and the errors attributed by various recent authors to Darcy appear upon closer inspection to have been those committed by the authors themselves.

* * *

In Paris in the year 1856 there was published by Victor Dalmont as a part of the *Libraire des Corps Impériaux des Ponts et Chaussées et des Mines* a monograph by the French engineer Henry Darcy [1], Inspector General of Bridges and Roads, bearing the title:

« LES FONTAINES PUBLIQUES
DE LA VILLE DE DIJON
Exposition et Application
DES PRINCIPES A SUIVRE ET DES FORMULES A EMPLOYER
Dans les Questions
de
DISTRIBUTION D'EAU
Ouvrage terminé
par un Appendice Relatif aux Fournitures d'Eau de Plusieurs Villes
AU FILTRAGE DES EAUX
et
A la Fabrication des Tuyaux de Fonte, de Plomb, de Tole et de Bitume »

For several years previously M. Henry Darcy had been engaged in modernizing and enlarging the public water works of the town of Dijon, and this treatise, comprising a 647-page volume of text and an accompanying *Atlas* of illustrations, constitutes an engineering report on that enterprise.

The item of present interest represents only a detail of the general work and appears in an appendix on pages 590 to 594 under the heading « Determination of the Law of Flow of Water Through Sand », and pertains to a

problem encountered by Darcy in designing a suitable filter for the system. Darcy needed to know how large a filter would be required for a given quantyti of water per day and, unable to find the desired information in the published literature, he proceeded to obtain it experimentally.

A drawing of the apparatus used is given as Figure 3 in the *Atlas* and is here reproduced in facsimile as Figure 1. This consisted of a vertical iron pipe, 0.35 meters in diameter and 3.50 meters in length (the figure shows 3.50 meters but the text says 2.50), flanged at both ends. At a height of 0.20 meters above the base of the column, there was placed a horizontal screen supported by an iron grillwork upon which rested a column, a meter or so in length, of loose sand. Water could be admitted into the system by means of a pipe, tapped into the column near its top, from the building water supply, and could be discharged through a faucet from the open chamber near its bottom. The faucet discharged into a measuring tank 1 meter square

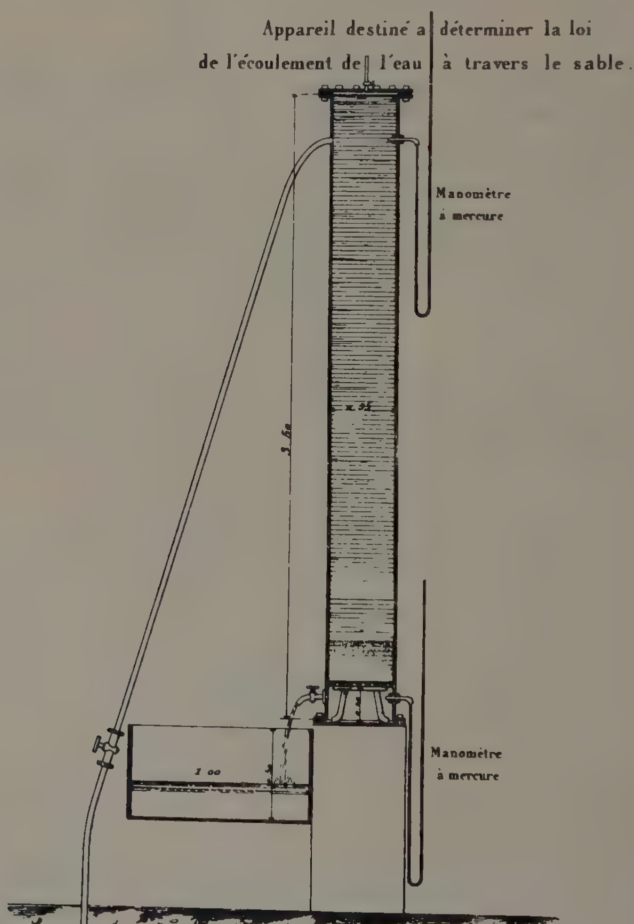

FIG. 1 — Fascimile of Darcy's illustration of his experimental apparatus. (From *Les Fontaines Publiques de la Ville de Dijon*, Atlas, Figure 3)

and 0.50 meter deep, and the flow rate could be controlled by means of adjustable valves in both the inlet pipe and the outlet faucet.

For measuring the pressures mercury manometers were used, one tapped into each of the open chambers above and below the sand column. The unit of pressure employed was the *meter of water* and all manometer readings were reported in meters of water measured above the bottom of the sand which was taken as an elevation datum. The observations of the mercury manometers were accordingly expressed directly in terms of the heights of the water columns of equivalent water manometers above a standard datum.

The experiments comprised several *series* of observations made between October 29 and November 2, 1855, and some additional experiments made during February 18 to 19, 1856. For each series the system was charged with a different sand and completely filled with water.

By adjustment of the inlet and outlet valves the water was made to flow downward through the sand at a series of successively increasing rates. For each rate a reading of the manometers was taken and recorded as a pressure difference in meters of water above the bottom of the sand. The results of two of these series, using different sands, are shown graphically in Figure 2. In each instance it will be seen that the total rate of discharge increases linearly with the drop in head across the sand of the two equivalent water manometers.

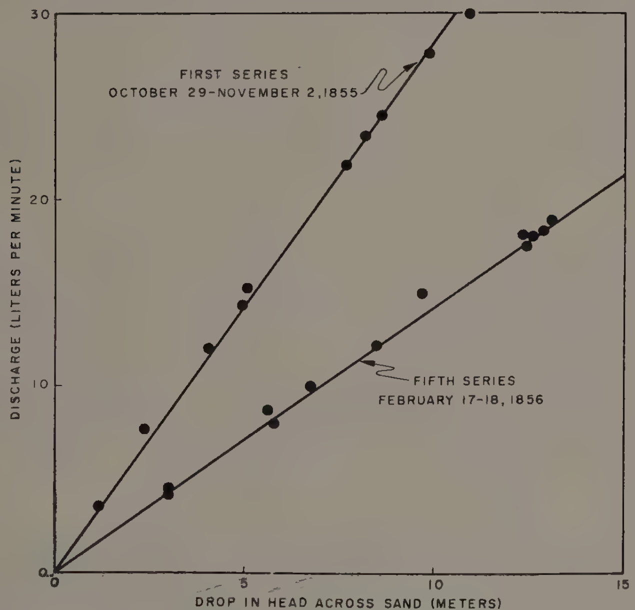

FIG. 2 — Graphs compiled from Darcy's tabular data on his experiments of October 29 to November 2, 1955, and of February 17-18, 1856, showing linear relation between flow rate and differences in heigts of equivalent water manometers.

Darcy's own summary of the results of his experiments is given in the following passage (p. 594):

« Ainsi, en appelant *e* l'épaisseur de la couche de sable, *s* sa superficie, P la pression atmosphérique, *h* la hauteur de l'eau sur cette couche, on aura

$P + h$ pour la pression à laquelle sera soumise la base supérieure; soient, de plus, $P \pm h_0$ la pression supportée par la surface inférieure, k un coefficient dépendant de la perméabilité de la couche, q le volume débité, on a $q = k \frac{s}{e} [h + e \mp h_0]$ qui se réduit à $q = k \frac{s}{e} (h + e)$ quand $h_0 = 0$, ou lorsque la pression sous le filtre est égale à la pression atmosphérique.

« Il est facile de déterminer la loi de décroissance de la hauteur d'eau h sur le filtre; en effet, soit dh la quantité dont cette hauteur s'abaisse pendant un temps dt, sa vitesse d'abaissement sera $-\frac{dh}{dt}$; mais l'équation ci-dessus donne encore pour cette vitesse l'expression

$$\frac{q}{s} = v = \frac{k}{e}(h + e).$$

« On aura donc $-\frac{dh}{dt} = \frac{k}{e}(h + e)$; d'où $\frac{dh}{(h + e)} = -\frac{k}{e} dt$,

et
$$l\,(h + e) = C - \frac{k}{e} t.$$

[l is the logarithm to the base e.]

« Si la valeur h_0 correspond au temps t_0 et h à un temps quelconque t, il viendra

$$l(h + e) = l(h_0 + e) - \frac{k}{e} [t - t_0] \qquad (1)$$

« Si on remplace maintenant $h + e$ et $h_0 + e$ par $\frac{qe}{sk}$ et $\frac{q_0 e}{sk}$, il viendra

$$lq = lq_0 - \frac{k}{e} (t - t_0) \qquad (2)$$

et les deux équations (1) et (2) donnent, soit la loi d'abaissement de la hauteur sur le filtre, soit la loi de variation des volumes débités à partir du temps t_0.

« Si k et e étaient inconnus, on voit qu'il faudrait deux expériences préliminaires pour faire disparaître de la seconde le rapport inconnu $\frac{k}{e}$. »

Translating Darcy's statements into the notation which will subsequently be used in the present paper, what Darcy found and stated was that, when water flows vertically downward through a sand, the volume of water Q passing through the system in unit time is given by

$$Q = KA \frac{h_1 - h_2}{l}, \text{ or by } - KA \frac{h_2 - h_1}{l}; \qquad (1)$$

and the volume crossing unit area in unit time by

$$Q/A = q = K \frac{h_1 - h_2}{l}, \text{ or by } - K \frac{h_2 - h_1}{l}, \qquad (2)$$

where K is a factor of proportionality, A the area of cross section and l the thickness of the sand, and h_1 and h_2 the heights above a standard reference elevation of water in equivalent water manometers terminated above and below the sand, respectively.

Writing equation (2) in differential from gives

$$q = - K\,(dh/dl). \qquad (3)$$

Soon after the publication of Darcy's account of these experiments, the

relationship expressed by equations (1) to (3) became known, appropriately, as Darcy's law.

It has subsequently come to be universally acknowledged that Darcy's law plays the same role in the theory of the conduction of fluids through porous solids as Ohm's law in the conduction of electricity, or of Fourier's law in the conduction of heat. On the other hand, Darcy's own statement of the law was in an empirical form which conveys no insight into the physics of the phenomenon. Consequently, during the succeeding century many separate attempts were made to give the statement of the law a more general and physically satisfactory form, with the result that there appeared in the technical literature a great variety of expressions, many mutually contradictory, but all credited directly or indirectly to Henry Darcy.

It has accordingly become recently the fashion, when some of these expressions have been found to be physically untenable, to attribute the error to Darcy himself. In fact one recent author, in discussing a supposed statement of Darcy's law which is valid for horizontal flow only, has gone so far as to explain that Darcy was led to the commission of this error by restricting his experiments to flow in a horizontal direction.

On this centennial occasion of Darcy's original publication, it would appear to be fitting, therefore, instead of merely paying our respects to Darcy in the form of an empty homage, that we first establish unequivocally what Darcy himself did and said with respect to the relationship which bears his name; second, try to ascertain the generality and physical content of the relationship and to give it a proper physical expression; and third, attempt to see how this fits into a general field theory of the flow of fluids through porous solids in three-dimensional space.

The first of these objectives has already been accomplished; the second and third will now be given our attention.

The Physical Content of Darcy's Law

As we have seen heretofore, what Darcy determined was that, when water flows vertically downward through a sand, the relation of the volume of water crossing unit area normal to the flow direction in unit time, to the thickness of the sand, and to the difference in heights of equivalent water manometers terminated above and below the sand, is given by the following equation:

$$q = \mathrm{K}\,\frac{h_1 - h_2}{l}\,, \text{ or } q = -\mathrm{K}\,\frac{h_2 - h_1}{l}\,, \qquad (2)$$

where K is «a coefficient depending upon the permeability of the sand».

Questions immediately arise regarding the generality of this result. Would it still be true if the water flowed upward through the sand? or horizontally? What changes would be effected in the relationship if some different liquid characterized by a different density and viscosity were used? In what manner does K depend upon the permeability of the sand, or upon its measurable statistical parameters such as coarseness and shape? And finally, what physical expression can be found which properly embodies all of these variables?

The answer to most of these questions can be determined empirically by an extension of Darcy's original experiment. If, for example, we construct an apparatus such as that shown in Figure 3, consisting of a movable cylinder with a rigid sand pack into which two manometers, at an axial distance l

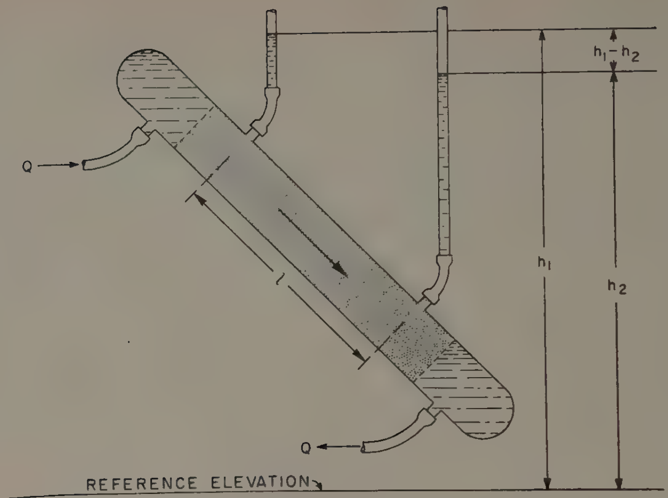

FIG. 3 — Apparatus for verifying Darcy's law for flow in various directions.

apart, are connected by flexible rubber tubing, we can determine the validity of Darcy's law with respect to the direction of flow. With the apparatus vertical and the flow downward at a total rate Q, the manometer difference $h_2 - h_1$ will have some fixed value Δh. Now, keeping Q constant and inverting the column so that the flow will be vertically upward, it will be found that Δh also remains constant. Next, setting the column horizontal, Δh still remains constant. In this manner we easily establish that Darcy's law is invariant with respect to the direction of the flow in the earth's gravity field, and that for a given Δh the flow rate Q remains constant whether the flow be in the direction of gravity or opposed to it, or in any other direction in three-dimensional space.

This leads immediately to a generalization for flow in three-dimensional space. At each point in such space there must exist a particular value of a scalar quantity h, defined as the height above a standard elevation datum of the water column in a manometer terminated at the given point. The ensemble of such values then gives rise to a scalar field in the quantity h with its attendant family of surfaces, h = constant. In such a scalar field water will flow in the direction perpendicular to the surfaces, h = constant, and at a rate given by

$$\vec{q} = -\mathrm{K} \operatorname{grad} h . \tag{4}$$

Continuing our empirical experimentation, we find that when we change either of the fluid properties, density or viscosity, or the geometrical properties of the sand, equation (4) still remains valid but the value of K changes. In particular, by varying one factor at a time, we find

$$\left.\begin{aligned} \mathrm{K} &\propto \rho \ , \\ \mathrm{K} &\propto 1/\mu \ , \end{aligned}\right\} \tag{5}$$

where ρ is the density and μ is the viscosity of the fluid.

Likewise, if we use a number of geometrically similar sands which differ only in grain size, we find that

$$\mathrm{K} \propto d^2 , \tag{6}$$

where d is a length such as the mean grain diameter, which characterizes the size scale of the pore structure of the sand.

Introducing the results of equations (5) and (6) into equation (4) then gives

$$\vec{q} = K' d^2(\rho/\mu)(-\operatorname{grad} h)\,, \tag{7}$$

in which K′ is a new factor of proportionality containing all other variables not hitherto explicitly evaluated. This remains, however, an empirical equation devoid of dynamical significance since there is no obvious reason why the flow of a viscous fluid through a porous solid should be proportional to a dimensionless quantity, $-\operatorname{grad} h$.

This deficiency can be eliminated when we introduce the equation relating the manometer height h to the dynamical quantities, gravity and pressure. At any point P within the flow system, characterized by elevation z and manometer height h, the pressure is given by the hydrostatic equation

$$p = \rho g\,(h - z)\,,$$

from which

$$h = (p/\rho g) + z\,, \tag{8}$$

and

$$-\operatorname{grad} h = -(1/\rho g)\operatorname{grad} p - \operatorname{grad} z\,. \tag{9}$$

Multiplying both sides of equation (9) by g then gives

$$-g \operatorname{grad} h = -(1/\rho)\operatorname{grad} p - g \operatorname{grad} z\,. \tag{10}$$

With the z-axis vertical and positive upward, then grad z is a unit vector directed upward, so that $-g \operatorname{grad} z$ is a vector of magnitude g directed downward. Designating this by $\vec{g}$, equation (10) becomes

$$-g \operatorname{grad} h = \vec{g} - (1/\rho)\operatorname{grad} p\,, \tag{11}$$

in which each of the terms to the right represents, both in direction and magnitude, the force exerted upon unit mass of the fluid by gravity and by the gradient of the fluid pressure respectively; the fluid flows in the direction of, and at a rate proportional to, their resultant, $-g \operatorname{grad} h$. Hence the dynamical factor g has evidently been concealed in the original factor K and must still be present in the residual factor K′. Introducing this explicitly, we may now write

$$\vec{q} = (Nd^2)(\rho/\mu)(-g \operatorname{grad} h)\,, \tag{12}$$

in which N is a final factor of proportionality. Dimensional inspection shows that N is dimensionless, and a physical review indicates that no dynamical variables have been omitted, so that N must be related to the only remaining variable, namely the shape of the passages through which the flow occurs. Since shape is expressed by angular measurement and angles are dimensionless, [L/L], then N, must be a dimensionless shape factor whose value is constant for systems which are either identically, or statistically, similar geometrically. By identical similarity is meant similarity in the strict Euclidean sense: all corresponding angles equal, and all corresponding lengths proportional. By statistical similarity is meant that two complex geometrical systems which may not be identically similar on a microscopic scale are still indistinguishable as to shape on a macroscopic scale — for example, two sets of randomly packed uniform spheres.

The term $(-g \operatorname{grad} h)$ can also be written in the form

$$-g \operatorname{grad} h = -\operatorname{grad}(gh)\,, \tag{13}$$

where gh represents the amount of work required to lift a unit mass of water

from the standard datum of elevation outside the system to the height h of the water in the manometer. Then, since the additional work required to transport the water down the waterfilled tube of the manometer to its terminus is zero, it follows that

$$gh = \Phi \tag{14}$$

is a measure of the energy per unit mass, or the *potential*, of the water in the system at the point at which the manometer is termintaed. A manometer is thus seen to be a fluid potentiometer, the potential at every point being linearly related to the manometer height h by equation (14). Then, if we let $\vec{E}$ be the force per unit mass, or the intensity of the force field acting upon the fluid, we have

$$\vec{E} = -\operatorname{grad}\Phi = -g \operatorname{grad} h = \vec{g} - (1/\rho) \operatorname{grad} p\,, \tag{15}$$

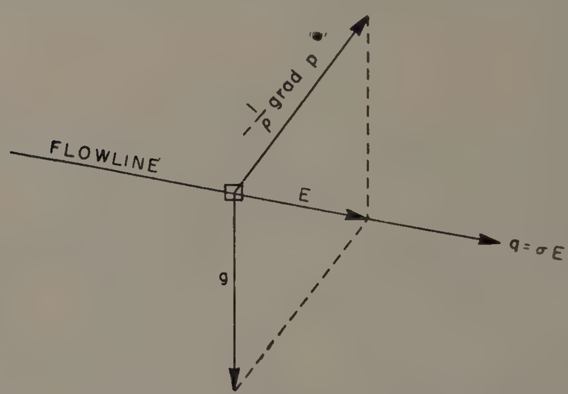

FIG. 4 — Relation of force per unit mass, $\vec{E}$, to the primary forces $\vec{g}$ and $-(1/p)$ grad p; and of the flow vector $\vec{q}$ to $\vec{E}$.

and, by substitution into equation (12), Darcy's law may be expressed in any of the following equivalent forms (Fig. 4):

$$\left.\begin{aligned} \vec{q} &= -(Nd^2)(\rho/\mu) \operatorname{grad}\Phi = \sigma\vec{E}\,, \\ \vec{q} &= -(Nd^2)(\rho/\mu)\, g \operatorname{grad} h = \sigma[\vec{g} - (1/\rho) \operatorname{grad} p]\,, \\ \vec{q} &= -(Nd^2)(\rho/\mu)\, g \operatorname{grad} h = (\sigma/\rho)[\rho\vec{g} - \operatorname{grad} p]\,, \end{aligned}\right\} \tag{16}$$

where

$$\sigma = [(Nd^2)(\rho/\mu)]$$

is the volume conductivity fo the system. In the last of equations (16), the bracketed term

$$[\rho\vec{g} - \operatorname{grad} p] = \rho\vec{E} = \vec{H} \tag{17}$$

represents the force $\vec{H}$ per unit volume.

When we compare Darcy's law in the form:

$$\vec{q} = \sigma\vec{E} = -\sigma \operatorname{grad}\Phi\,,$$

with Ohm's law:

$$\vec{i} = \sigma_e \vec{E}_e = -\sigma_e \operatorname{grad} V,$$

where $\vec{i}$ is the current density, σ_e the electrical conductivity, $\vec{E}_e$ the electrical force-field intensity, and V the electrical potential, the physical as well as the mathematical analogy between Darcy's law and Ohm's law becomes immediately apparent.

The problem of permeability. — Now that we have achieved a complete physical statement of Darcy's law, it remains for us to define what shall be meant by the permeability of the system.

It will be recalled that Darcy stated that the factor K is «a coefficient depending upon the permeability of the sand » Following this it has often been the custom, especially among ground-water hydrologists, to define the permeability of a system to be synonymous with K. But, as we have seen, the factor K is the lumped parameter,

$$K = (Nd^2)(\rho/\mu)\, g,$$

comprising the geometrical properties of the sand, the dynamical properties of the fluid, and even the acceleration of gravity. Consequently, if K is taken as a measure of the permeability, it will be seen that the same sand will have different permeabilities to different fluids.

During recent years there has been a convergence of opinion toward the conclusion that permeability should be a constant of the solid independently of the fluids involved. If we accept this view, then it is seen that the only property of the solid affecting the rate of flow is the geometrical factor,

$$k \equiv Nd^2,$$

which we may accordingly define to be its permeability. Then, since N is dimensionless and d is a length, it follows that the dimensions of permeability are $[L^2]$; and in any consistent system of units, the unit of permeability is the square of the unit of length.

In practice the magnitude of this quantity, for a given porous solid, is determined hydrodynamically by flowing a liquid through the solid, and measuring all variables except k, and then solving Darcy's law for k:

$$k = Nd^2 = \frac{\overline{q}\mu}{\rho\vec{g} - \operatorname{grad} p},$$

which, when the vector quantities are resolved into their components in the flow direction s, becomes

$$k = \frac{q\mu}{\rho g_s - \partial p/\partial s}. \tag{18}$$

It is found in this manner that for randomly packed, uniform spheres of diameter d, the value of the shape-factor N is approximately 6×10^{-4}. Then, for a pack of uniform spheres of any size, the permeability will be approximately

$$k = (6 \times 10^{-4})\, d^2.$$

If d, for different packs, is allowed to vary from about 10^{-4} to 10^{-1} cm, corresponding to the approximate range of grain sizes from fine silts to coarse sands, the permeability will vary from about 10^{-12} to 10^{-6} cm^2, which is also approximately the range of the permeabilities of the corresponding clastic sediments.

In view of the fact that magnitudes of permeabilities of rocks are remote from that of the square of any unit of length in commun use, there is some

advantage in having a practical unit such that most measured values fall within the range 1 — 10,000 practical units. If such a practical unit is to fit into a consistent system of measurement without awkward conversion factors, then it must also be a submultiple of the fundamental unit of the form:

1 practical unit $= 10^{-n}$ fundamental units.

In the cgs system with the fundamental unit the (centimeter)^2, the optimum value of the exponent n would be about, 12 or

$$1 \text{ practical unit} = 10^{-12} \text{ cm}^2 .$$

Regrettably the unit of permeability used almost universally in the petroleum industry, for which the name «darcy» has been preempted, was defined originally in terms both of an incomplete statement of Darcy's law:[2,3]

$$k = -\frac{q\mu}{\partial p/\partial s}, \tag{19}$$

and an inconsistent system of measurement. The permeability k is defined to be 1 darcy when $q = 1\,(\text{cm}^3/\text{cm}^2)/\text{sec}$, $\mu = 1$ centipoise, and $\partial p/\partial s = 1$ *atmosphere*/cm.

In the complete Darcy's law of equation (18), the factor ρg_s, except when the motion is horizontal, is of comparable magnitude to $\partial p/\partial s$, and so cannot be ignored. Consequently, since it is not practical to measure ρg_s in atmospheres/cm, it follows that permeabilities, expressed in darcys, cannot be used in a proper statement of Darcy's law without the insertion of a numerical factor to convert ρg_s from cgs units into atmospheres/cm.

The only alternative is to convert permeabilities expressed in darcys into the cgs unit, the cm^2. For this conversion

$$1 \text{ darcy} = 0.987 \times 10^{-8} \text{ cm}^2 ,$$

which is within 1.3 percent of the submultiple, 10^{-8} cm^2.

Few permeability measurements are accurate to within 1.3 percent and, when several specimens from the same formation are measured, the scatter is much greater than this. Consequently, for all ordinary computations, the approximate conversions:

$$1 \text{ darcy} \cong 10^{-8} \text{ cm}^2 ,$$
$$1 \text{ millidarcy} \cong 10^{-11} \text{ cm}^2 ,$$

are more accurate than the permeability data available; though if the data warrant it, the more precise conversion can of course be used.

Derivation of Darcy's Law from Navier-Stokes Equation

Having thus achieved the desired generalization and a proper physical statement of Darcy's law by an extension of the empirical method which Darcy himself employed, let us now see if the same result can be derived directly from the fundamental equation of Navier and for the motion of a viscous fluid.

Macroscopic and microscopic scales.—In order to do this we must first distinguish between the two size scales, the macroscopic and the microscopic, on which the phenomena considered are to be viewed.

The macroscopic scale, which is the one we have been using thus far, is a scale that is large as compared with the grain or pore size of the porous solid. On this scale the flow of a fluid through a porous solid is seen as a continuous phenomenon in space. However, when we are dealing with macroscopic quantities which have particular values at each point in space,

but which may vary with position, it is necessary for us to define more clearly what is meant by the value of a macroscopic quantity at a given point.

This can be illustrated with the concept of porosity. Suppose that we are interested in the porosity at a particular point. About this point we take a finite volume element ΔV, which is large as compared with the grain or pore size of the rock. Within this volume element the average porosity is defined to be

$$\bar{f} = \frac{\Delta V_f}{\Delta V}, \tag{20}$$

where ΔV_f is the pore volume within ΔV. We then allow ΔV to contract about the point P and note the value of $\bar{f}$ as ΔV diminishes. If we plot $\bar{f}$ as a function of ΔV (Fig. 5), it will approach smoothly a limiting value as ΔV diminishes until ΔV approaches the grain or pore size of the solid. At this stage $\bar{f}$ will begin to very erratically and will ultimately attain the value of either 1 or 0, depending upon whether P falls within the void or the solid space.

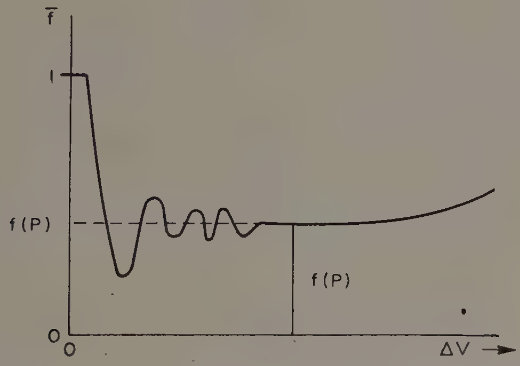

FIG. 5 — Method of defining point values of macroscopic quantities illustrated with the porosity f.

However, if we extrapolate the smooth part of the curve of $\bar{f}$ versus ΔV to its limit as ΔV tends to zero, we shall obtain an unambiguous value of f at the point P. We thus define the value of the porosity f at the point P to be

$$f(P) = \underset{\Delta V \to 0}{\text{extrap lim}} \frac{\Delta V_f}{\Delta V}, \tag{21}$$

where *extrap lim* signifies the extrapolated limit as obtained in the manner just described.

By an analogous operation the point value of any other macroscopic quantity may be obtained, so that hereafter, when such quantities are being considered, their values will be understood to be defined in the foregoing manner, and we may state more simply:

$$
\left.
\begin{aligned}
Q(P) &= \lim_{\Delta V \to 0} \bar{Q}(\Delta V), \\
&\text{or} \\
Q(P) &= \lim_{\Delta S \to 0} \bar{Q}(\Delta S), \\
&\text{or} \\
Q(P) &= \lim_{\Delta s \to 0} \bar{Q}(\Delta s),
\end{aligned}
\right\} \tag{22}
$$

where the quantity of interest is a function of a volume, an area, or a length, respectively.

The microscopic scale, on the contrary, is a scale commensurate with the grain or pore size of the solid, but still large as compared with molecular dimensions or of the motional irregularities due to Brownian or molecular movements.

Microscopic equations of motion.—Let us next consider the steady, macroscopically rectilinear flow of an incompressible fluid through a porous solid which is macroscopically homogeneous and isotropic with respect to porosity and permeability. We shall then have the fluid flowing with a constant macroscopic flow rate $\vec{q}$ under a constant impelling force per unit of mass $\vec{E}$, and in virtue of the isotropy of the system, we shall have

$$\vec{q} = \sigma \vec{E}, \tag{23}$$

where σ is an unknown scalar whose value we shall seek to determine.

Then, choosing x-, y-, and z-axes,

$$
\left.
\begin{aligned}
\vec{q} &= \vec{i} q_x + \vec{j} q_y + \vec{k} q_z, \\
\vec{E} &= \vec{i} E_x + \vec{j} E_y + \vec{k} E_z,
\end{aligned}
\right\} \tag{24}
$$

and

$$
\left.
\begin{aligned}
q_x &= \sigma E_x, \\
q_y &= \sigma E_y, \\
q_z &= \sigma E_z.
\end{aligned}
\right\} \tag{25}
$$

where $\vec{i}$, $\vec{j}$, and $\vec{k}$ are unit vectors parallel to the x-, y-, and z-axes, respectively, and the subscripts signify the corresponding scalar components of the vectors

Further, there will be no loss of generality, and our analysis will be some what simplified, if we choose the x-axis in the macroscopic direction of flow. Then

$$
\left.
\begin{aligned}
&\vec{q} = \vec{i} q_x; \ \vec{E} = \vec{i} E_x, \\
\text{and} \quad &q_y = q_z = 0; \ E_y = E_z = 0.
\end{aligned}
\right\} \tag{26}
$$

Next, consider the microscopic flow through a macroscopic volume element ΔV of sides Δx, Δy, and Δz. The void space in such an element will be seen to be an intricately branching, threedimensional network of flow channels, each of continuously varying cross section. A fluid particle passing through such a system will follow a continuously curving tortuous path. Moreover, the speed of the particle will alternately increase and decrease as the cross section of the channel through which it flows becomes larger or smaller. Such a particle will accordingly be seen to be in a continuous state

of acceleration with the acceleration vector free to assume any possible direction in space.

Consider now the forces which act upon a small volume element dV of this fluid. By Newton's second law of motion

$$dm\,\vec{a} = \Sigma d\vec{F}\,, \tag{27}$$

where dm is the mass of the fluid, $\vec{a}$ the acceleration, and $\Sigma d\vec{F}$ is the sum of all the forces acting upon the fluid contained within dV. There are many ways in which these forces may be resolved, but for present purposes it will be convenient to resolve them into a driving or impelling force $d\vec{F}_d$ and a resistive force arising from the viscous resistance of the fluid element to deformation, $d\vec{F}_v$. Equation (27) then becomes

$$dm\,\vec{a} = d\vec{F}_d + d\vec{F}_v\,. \tag{28}$$

By the principle of D'Alembert we may also introduce a force $d\vec{F}_a = -\,dm\,\vec{a}$, which is the inertial reaction of the mass dm to the acceleration $\vec{a}$, and with this substitution equation (28) becomes

$$d\vec{F}_d + d\vec{F}_a + d\vec{F}_v = 0\,. \tag{29}$$

Of these forces, $d\vec{F}_d$, which is imposed from without and does not depend primarily upon the motion of the fluid, may be regarded as the independent variable. The forces $d\vec{F}_a$ and $d\vec{F}_v$ both owe their existences to the fluid motion, and their effect is to impede that motion.

The relation of the separate terms of equation (29) to the externally applied forces and the fluid motion are given by the equatoin of Navier and Stokes, which in vector form may be written as follows:

$$\rho[\vec{g} - (1/\rho)\,\mathrm{grad}\,p]\,dV = \rho\,(D\vec{v}/Dt)\,dV - \mu\,[\nabla^2\vec{v} + (1/3)\,\nabla\nabla\,.\,\vec{v}]\,dV\,, \tag{30}$$

in which $\vec{v}$ is the microscopic velocity, and $p = -(1/3)(\sigma_x + \sigma_y + \sigma_z)$ is the microscopic pressure at a point. The σ's are normal components of microscopic stress.

The expression $D\vec{v}/Dt$ is the total derivative with respect to time of the velocity $\vec{v}$, and is equal to the acceleration $\vec{a}$. This can be expanded into

$$D\vec{v}/Dt = \frac{\partial \vec{v}}{\partial t} + \vec{v}\,.\nabla\vec{v}\,,$$

in which the term $\partial\vec{v}/\partial t$ signifying the rate of change of the velocity at a particular point is zero for steady motion. The expression $\nabla.\,\vec{v}$, in the last term of equation (30), is the divergence of the velocity; and for an incompressible fluid this also is zero.

Since the flow being considered is the steady motion of an incompressible fluid, equation (30) simplifies to

$$\rho[\vec{g} - (1/\rho)\,\mathrm{grad}\,p]\,dV = \rho\,(\vec{v}\,.\,\nabla\vec{v})\,dV - \mu(\nabla^2\vec{v})\,dV\,. \tag{31}$$

Transformation from microscopic to macroscopic equations of motion.— Equation (31) expresses the relation of the fluid velocity and its derivatives in a small microscopic volume element to the applied force $d\vec{F}_d$ acting upon that element. If we could integrate the three terms of this equation with respect to the volume, over the macroscopic volume ΔV, and then convert the results into equivalent macroscopic variables, our problem would be solved.

In fact the integration of the first two terms presents no difficulty. The total driving force on the fluid content of the volume ΔV, as obtained by integrating the microscopic forces $d\vec{F}_d$, is:

$$\vec{F}_d = \int d\vec{F}_d = \int_{f\Delta V} (\rho\vec{g} - \text{grad}\, p)\, dV = (\rho\vec{g} - \overline{\text{grad}\, p})\, f\Delta V\,, \qquad (32)$$

where $\overline{\text{grad}\, p}$ is the volumetric average of the microscopic grad p over the fluid volume $f\Delta V$.

From the macroscopic equations, the driving force $\vec{F}_d$ is given by:

$$\vec{F}_d = \rho f\Delta V\, \vec{E} = \rho[\vec{g} - (1/\rho)\,\text{grad}\, p]\, f\Delta V\,. \qquad (33)$$

Then by combining equations (32) and (33),

$$\int_{f\Delta V} (\rho\vec{g} - \text{grad}\, p) dV = (\rho\vec{g} - \overline{\text{grad}\, p}) f\Delta V = \rho f\Delta V\, [\vec{g} - (1/\rho)\,\text{grad}\, p]\,, \qquad (34)$$

from which it is seen that the macroscopic grad p is equal to the volumetric average, $\overline{\text{grad}\, p}$, of the microscopic grad p.

Integrating the inertial term:

$$\left.\begin{aligned}
\vec{F}_a = \int d\vec{F}_a &= \rho \int_{f\Delta V} \vec{v} \cdot \nabla\vec{v}\, dV \\
&= \vec{i}\,\rho \int_{f\Delta V} (u\,\partial u/\partial x + v\,\partial u/\partial y + w\,\partial u/\partial z)\, dx\, dy\, dz \\
&+ \vec{j}\,\rho \int_{f\Delta V} (u\,\partial v/\partial x + v\,\partial v/\partial y + w\,\partial v/\partial z)\, dx\, dy\, dz \\
&+ \vec{k}\,\rho \int_{f\Delta V} (u\,\partial w/\partial x + v\,\partial w/\partial y + w\,\partial w/\partial z)\, dx\, dy\, dz\,.
\end{aligned}\right\} \qquad (35)$$

Since there is no net gain in velocity with macroscopic distance, each separate integral of the expanded form of equation (35) is equal to zero, and we obtain for the volume element ΔV,

$$\vec{F}_a = \int d\vec{F}_a = 0\,. \qquad (36)$$

Then, in virtue of equation (36),

$$\int d\vec{F}_v = -\mu \int_{f\Delta V} \nabla^2\vec{v}\, dV = -\vec{F}_d. \qquad (37)$$

Ordinarily the evaluation of

$$\int_{f\Delta V} \nabla^2\vec{v}\, dV$$

would require a detailed consideration of the geometry of the void space through which the flow occurs and of the flow field within that space. This difficulty can be circumvented, however, and the integral evaluated except for a dimensionless factor of proportionality, *provided the flow field is kinematically similar for different rates of flow.*

Criteria of similarity. Consider two flow systems consisting of two geometrically similar porous solids through which two different fluids are flowing. The criterion of geometrical similarity is that if l_1 and l_2 are any

corresponding lengths of the two systems, then for every pair of such lengths

$$l_2/l_1 = l_r = \text{const.} \tag{38}$$

The criterion of kinematic similarity is that if $\vec{v}_1$ and $\vec{v}_2$ are the velocities at corresponding points in the two systems, the two velocities must have the same direction and their magnitudes the ratio

$$v_2/v_1 = v_r = \text{const}\,.$$

Then, since the forces $d\vec{F}_a$ and $d\vec{F}_v$ acting upon a fluid element are each determined by the velocities and the fluid density, or viscosity, and $d\vec{F}_d$ is determined by $d\vec{F}_a$ and $d\vec{F}_v$, if the fluid motions of the two systems are kinematically similar, all corresponding forces will have the same directions and their magnitudes the same ratio (Fig. 6).

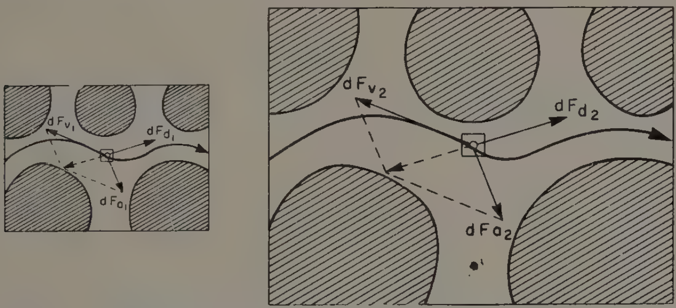

FIG. 6 — Microscopic views of two dynamically similar flow systems

Thus

$$\frac{(dF_a)_2}{(dF_a)_1} = \frac{(dF_v)_2}{(dF_v)_1} = \frac{(dF_d)_2}{(dF_d)_1}\,. \tag{39}$$

Since only two of the three forces are independent, we need to consider the ratios of only the first two, and, by reciprocation,

$$\left(\frac{dF_a}{dF_v}\right)_2 = \left(\frac{dF_a}{dF_v}\right)_1, \tag{40}$$

indicating that for each system the ratio of the inertial to the viscous force must be the same.

From equation (31)

$$\frac{(dF_a)_2}{(dF_a)_1} = \frac{\rho_2}{\rho_1}\,\frac{(\vec{v}\,.\,\nabla\vec{v})_2}{(\vec{v}\,.\,\nabla\vec{v})_1}\,\frac{dV_2}{dV_1}\,; \tag{41}$$

$$\frac{(dF_v)_2}{(dF_v)_1} = \frac{\mu_2}{\mu_1}\,\frac{(\nabla^2\vec{v})_2}{(\nabla^2\vec{v})_1}\,\frac{dV_2}{dV_1}\,. \tag{42}$$

In equation (41) $\vec{v}\,.\,\nabla\vec{v}$ expands into the sum of a series of terms, each of the form $u\,\partial u/\partial x$, which is a velocity squared divided by a length. In equation (42) $\nabla^2\vec{v}$ expands into a series of terms, each of the form $\partial^2 u/\partial x^2$,

which is a velocity divided by the square of a length. Then, since the ratios of all corresponding velocities and of all corresponding lengths of the two systems are constant, we may choose any suitable velocity and any convenient length. We accordingly choose for the velocity the macroscopic flow rate q whose dimensions are $[L^3 L^{-2} T^{-1}]$, or $[LT^{-1}]$. For the characteristic length we choose d, which must be some convenient statistical length parameter of the microscopic geometry of the system. With these substitutions the first two terms of equation (39) become

$$\frac{\rho_2 q_2^2 d_2^2}{\rho_1 q_1^2 d_1^2} = \frac{\mu_2 q_2 d_2}{\mu_1 q_1 d_1}, \tag{43}$$

and, by reciprocation, equation (40) becomes

$$\frac{\rho_2 q_2^2 d_2^2}{\mu_2 q_2 d_2} = \frac{\rho_1 q_1^2 d_1^2}{\mu_1 q_1 d_1},$$

or

$$\frac{q_2 d_2}{\mu_2/\rho_2} = \frac{q_1 d_1}{\mu_1/\rho_1}. \tag{44}$$

The dimensionless quantity $(qd)/(\mu/\rho)$ is the Reynolds number R of the system, which, as seen from its derivation, is a measure of the ratio of the inertial to the viscous forces of the system. Our criterion for kinematic similarity between the two systems thus reduces to the requirement that

$$R_2 = R_1. \tag{45}$$

Now let us specialize the two systems by making

$$d_2 = d_1, \ \rho_2 = \rho_1, \ \mu_2 = \mu_1,$$

which is equivalent to requiring the same fluid to flow through the same porous solid at velocities, q_2 and q_1. However, when these values are substituted into equation (44), we obtain

$$q_2 = q_1,$$

indicating that, in general, when the same fluid flows thourgh a given porous solid at two different rates, the resulting flow fields cannot be kinematically similar.

However, since dF_a is proportional to q^2 and dF_v to q, then as q is decreased dF_a diminishes much more rapidly than dF_v. Consequently there must be some limiting value of $q = q^*$, or of $R = R^*$, at and below which the inertial force dF_a is so much less than the viscous force dF_v that the effect of the former is negligible as compared with the latter. For flow in this domain we may then write

$$d\vec{F}_v = -d\vec{F}_d;$$

and the force ratios become

$$\frac{(dF_v)_2}{(dF_v)_1} = \frac{(dF_d)_2}{(dF_d)_1}, \tag{46}$$

whereby kinematical similarity is maintained for all rates of flow $q < q^*$.

Integration of viscous forces.—With this result established let us now return to the integration of $d\vec{F}_v$ over the volume element $f\Delta V$.

$$\int_{f\Delta V} d\vec{F}_v = -\mu\left[\vec{i}\int_{f\Delta V} \nabla^2 u\, dV + \vec{j}\int_{f\Delta V} \nabla^2 v\, dV + \vec{k}\int_{f\Delta V} \nabla^2 w\, dV\right]. \tag{47}$$

In virtue of the fact that, by our choice of axes, q_y and q_z are both zero

and there is no net flow in the y- or z-direction, the last two integrals to the right are both zero, and equation (47) simplifies to

$$\int_{f\Delta V} d\vec{F}_v = -\vec{i}\,\mu \int_{f\Delta V} \nabla^2 u \, dV \,. \tag{48}$$

From our earlier discussion, so long as the flow remains kinematically similar for different rates, the quantity $\nabla^2 u$, which is a velocity divided by the square of a length, is related to the macroscopic parameters by

$$\nabla^2 u = \alpha\,(q/d^2)\,, \tag{49}$$

where α is a dimensionless constant of proportionality for the element dV but has a different value for each different element of volume.

Substituting equation (49) into equation (48) then gives

$$\int_{f\Delta V} d\vec{F}_v = -\vec{i}\mu\,(q/d^2) \int_{f\Delta V} \alpha\, dV = -\frac{\mu \vec{q}}{Nd^2} f\Delta V\,, \tag{50}$$

where $1/N$ is the average value of α over $f\Delta V$.

Substituting this result into equation (37), we obtain

$$\rho[\vec{g} - (1/\rho)\,\mathrm{grad}\,p]\, f\Delta V = -\int d\vec{F}_v = \frac{\mu \vec{q}}{Nd^2} f\Delta V\,,$$

or

$$\vec{q} = Nd^2\,(\rho/\mu)\,[\vec{g} - (1/\rho)\,\mathrm{grad}\,p]\,, \tag{51}$$

which is the derived Darcy's law in the same form as equation (16) deduced earlier from empirical data.

Discussion of Darcy's law.—The direct derivation of Darcy's law from fundamental mechanics affords a further insight into the physics of the phenomena involved over what was obtainable from the earlier method of empirical experimentation. It has long been known empirically, for example, that Darcy's law fails at sufficiently high rates of flow, or at a Reynolds number, based on the mean grain diameter as the characteristic length, of the order of $R = 1$.[4]

At the same time one of the most common statements made about Darcy's law has been that it is a special case of Poiseuille's law; and most efforts at its derivation have been based upon various models of capillary tubes or of pipes. It also has been known since the classical studies of Osborne Reynolds[5] in 1883 that Poiseuille's law fails when the flow makes the transition from laminar to turbulent motion, so the conclusion most often reached as to the cause of the failure of Darcy's law has been that the motion has become turbulent.

From what we have seen, this represents a serious misinterpretation and lack of understanding of Darcy's law. In the Darcy flow each particle moves along a continuously curvilinear path at a continuously varying speed, and hence with a continuously varying acceleration; in the Poiseuille flow each particle moves along a rectilinear path at constant velocity and zero acceleration. Therefore, instead of Darcy's law being a special case of Poiseuille's law, the converse is true; Poiseuille's law is in fact a very special case of Darcy's law. Another special case of Darcy's law is the rectilinear flow between parallel plates.

Consequently deductions concerning the Darcy-type flow made from the simpler Poiseuille flow are likely to be seriously misleading. The deduction that the Reynolds number at which Darcy's law fails is also the one at

which turbulence begins is a case in point. We have seen that the cause of the failure of Darcy's law is the distortion that results in the flowlines when the velocity is great enough that the inertial force becomes significant. This occurs at a very slow creeping rate of flow which, for water, has the approximate value of

$$q^* \cong \frac{\mu}{\rho d} \cong (1 \times 10^{-2}\ \text{cm}^2\ \text{sec}^{-1})\ (1/d),$$

corresponding to $R^* = 1$, when d is the mean grain diameter.

Thus when $d = 10^{-2}$ cm Darcy's law fails at a flow rate $q \cong 1$ cm/sec.

Since this represents the threshold at which the effects of inertial forces first become perceptible, and since turbulence is the result of inertial forces becoming predominant with respect to resistive forces, it would be inferred that the incidence of turbulence in the Darcy flow would occur at very much higher velocities or at very much higher Reynolds numbers than those for which linearity between the flow rate and the driving force ceases. That this is in fact the case has been verified by visual observations of the flow of water containing a dilute suspension of colloidal bentonite through a transparent cell containing cylindrical obstacles. This system, when observed in polarized light, exhibits flow birefringence which is stationary for steady laminar flow but highly oscillatory when the motion is turbulent. Observations of only moderate precision indicate that the incidence of turbulence occurs at a Reynolds number of the order, of 600 or 700, or at a flow velocity of the order of several hundred times that at which Darcy's law fails.

Examination of the shape-factor N.—In both procedures used thus far, the factor N has emerged simply as a dimensionless factor of proportionality whose magnitude is a function of the statistical geometrical shape of the void space through which the flow occurs. For systems which are either identically or statistically similar geometrically, N has the same value. Beyond this we have little idea of the manner in which N is related to the shape or of what its numerical magnitude should be, except as may be determined by experiment. Let us now see if the value of N, at least to within an order of magnitude, can be determined theoretically.

Since we have already seen in equation (50) that $1/N = \bar{\alpha}$, where α is the factor of proportionality between the macroscopic quantity q/d^2 and the microscopic quantity $\nabla^2 u$, it follows that in order to determine the magnitude of N we must first determine that of the average value of $\nabla^2 u$. For this purpose, with the fluid incompressible, the macroscopic flow parallel to the x-axis, and the inertial forces negligible, only the x-component of the Navier-Stokes equation (31),

$$\rho g_x - \partial p/\partial x = -\mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right), \tag{52}$$

needs to be considered. When this is integrated with respect to the volume over the fluid space $f\Delta V$, it becomes

$$\overline{(\rho g_x - \partial p/\partial x)}\ f\Delta V = -\mu\left[\int_{f\Delta V} \frac{\partial^2 u}{\partial x^2}\,dV + \int_{f\Delta V} \frac{d^2 u}{\partial y^2}\,dV + \int_{f\Delta V} \frac{\partial^2 u}{\partial z^2}\,dV\right]. \tag{53}$$

Of the three integrals to the right, the first, which represents the expansion of the fluid in the x-direction, is zero; and from symmetry, the last two, both being the integrals of derivatives with respect to axes at right angles to the flow, are equal to each other. In consequence, equation (53) is reduced to

the simpler form:

$$(\overline{\rho g_x - \partial p/\partial x})f\Delta V = -2\mu\int(\partial^2 u/\partial y^2)\,dV \doteq -2\mu\,(\overline{\partial^2 u/\partial y^2})f\Delta V\,.$$

in which $\overline{\partial^2 u/\partial y^2}$ is the average value of $\partial^2 u/\partial y^2$ throughout the macroscopic volume element. Solving this for $\overline{\partial^2 u/\partial y^2}$ then gives

$$(\overline{\partial^2 u/\partial y^2}) = -(1/2\mu)(\overline{\rho g_x - \partial p/\partial x})\,. \tag{54}$$

Our problem now reduces to one of attempting to determine the average value of $\partial^2 u/\partial y^2$ for the system in terms of the kinematics of the flow itself. If we extend any line through the system parallel to the y-axis, this line will pass alternately through solid and fluid spaces. At each point on the line in the fluid space, there will be a particular value of the x-component u of the velocity, which will be zero at each fluid-solid contact, but elsewhere will have finite, and usually positive values, giving some kind of a velocity profile across each fluid gap. If this profile for each gap could be determined, then we could also compute $\partial^2 u/\partial y^2$ at each point on the line and thereby determine $\overline{\partial^2 u/\partial y^2}$ for the line, which, in a homogeneous and isotropic system, would also be the average value for a volume.

To attempt to do this in detail would be a statistical undertaking beyond the scope of the present paper. As a first approximation, however, we may simplify the problem by assuming:

1. That all the gaps are equal and of width $2\bar{\lambda}$, where $\bar{\lambda}$ is the average half-width of the actual gaps.

2. That through each gap the velocity profile satisfies the differential equation

$$\partial^2 u/\partial y^2 = \overline{\partial^2 u/\partial y^2} = -C\,. \tag{55}$$

3. That the total discharge through the averaged gaps is the same as that through the actual gaps.

The velocity profile for the averaged gaps can then be obtained by integrating equation (55) with respect to y. Taking a local origin of coordinates at the middle of the gap, and integrating equation (55) twice with respect to y, we obtain

$$u = -Cy^2/2 + Ay + B\,, \tag{56}$$

in which A and B are constants of integration. Then, supplying the boundary conditions, $u = 0$ when $y = \pm\bar{\lambda}$, gives

$$O = -C\bar{\lambda}^2/2 \pm A\bar{\lambda} + B$$

from which

$$A = 0;\ B = C\bar{\lambda}^2/2\,.$$

Substituting these into equation (56), we obtain

$$u = (C/2)(\bar{\lambda}^2 - y^2) \tag{57}$$

as the equation of the parabolic profile of the averaged velocity across the averaged gap.

The mean value, $\bar{u}$, of u across this gap is given by

$$\bar{u} = (1/\bar{\lambda})\int_0^{\bar{\lambda}} u\,dy = \frac{C}{2\bar{\lambda}}\int_0^{\bar{\lambda}}(\bar{\lambda}^2 - y^2)\,dy = C\bar{\lambda}^2/3\,. \tag{58}$$

Then, replacing C by $(1/2\mu)(\overline{\rho g_x - \partial p/\partial x})$ from equations (55) and (54), we obtain

$$\bar{u} = \frac{\bar{\lambda}^2}{6}\,\frac{\rho}{\mu}\,[\overline{g_x - (1/\rho)\partial p/\partial x}]\,. \tag{59}$$

This can be converted into terms of the macroscopic velocity, q_x, by noting that for a macroscopic length of line l normal to the flow direction

$$q_x l = 2\bar{u} \Sigma\lambda ,$$

or

$$q_x = 2\bar{u} (\Sigma\lambda/l) = \bar{u} f \tag{60}$$

where f is the porosity.

With this substitution equation (59) becomes

$$q_x = \frac{f\bar{\lambda}^2}{6} \cdot \frac{\rho}{\mu} \cdot [g_x - (1/\rho) \partial p/\rho x] , \tag{61}$$

or, more generally,

$$\vec{q} = \frac{f\bar{\lambda}^2}{6} \cdot \frac{\rho}{\mu} \cdot [\vec{g} - (1/\rho) \operatorname{grad} p] , \tag{62}$$

which is a statement of Darcy's law that is valid to the extent of the validity of the averaging approximation used in its derivation.

In this, it will be noted, that the geometrical factor $(f/6)\bar{\lambda}^2$ represents the permeability, so that

$$k \simeq (f/6)\bar{\lambda}^2 = N_\lambda \bar{\lambda}^2 ,$$

or

$$N_\lambda \simeq f/6 , \tag{63}$$

where N_λ is the shape factor corresponding to λ as the characteristic length of the system.

Determination of $\bar{\lambda}$.—The mean half gap-width $\bar{\lambda}$ along a linear traverse can be determined in either of two ways. The most obvious way is by direct observation by means of micrometer measurements along rectilinear traverses across a plane section of the porous solid.

Of greater theoretical interest, however, is an indirect method due to Corrsin [6]. Instead of a line, let a rectangular prism of cross-sectional area δ^2, where δ can be made arbitrarily small, be passed through a porous solid which is macroscopically homogeneous and isotropic. This prism will pass alternately through solid segments and void segments. Let n be the number of each which is traversed per unit length. Then the number of intersections with the solid surface per unit length will be $2n$, and if a is the average area of the solid surface cut out by the prism at each intersection, the total area per unit length, $d\beta$, will be

$$d\beta = 2n\,\bar{a} . \tag{64}$$

If a parallel family of such prisms is made to fill all space, the number per unit area perpendicular to the axis of the prisms will be $1/\delta^2$, and the solid surface intersected per unit volume will be

$$\beta = 2n\,(\bar{a}/\delta^2) . \tag{65}$$

In addition, the total length of the void spaces per unit length of line will be

$$2n\,\bar{\lambda} = f ,$$

or

$$n = f/(2\bar{\lambda}) . \tag{66}$$

Substituting the value of n from equation (66) into (65) and solving for λ then gives

$$\bar{\lambda} = \frac{f}{\beta} \cdot \frac{\bar{a}}{\delta^2} . \tag{67}$$

Since f and β can be measured, the value of $\bar{\lambda}$ could be determined if $\bar{a}/\delta^2$ were known.

The latter can be determined in the following manner: A homogeneous and isotropic distribution of the internal surface S, inside a macroscopic space, implies that if all equal elements dS of the surface were placed without rotation at the center of a reference sphere, their normals would intersect the sphere with a uniform surface density. Then, with the normals fixed in direction, if the surface elements were all moved equal radial distances outward, at some fixed radius they would coalesce to form the surface of a sphere. If the prisms of cross-sectional area δ^2, parallel to a given line, were then passed through this sphere, the average value of a/δ^2 would be

$$\overline{a/\delta^2} = \frac{\Sigma a}{\Sigma \delta^2}; \qquad (68)$$

and, when the summation includes one whole hemisphere,

$$\overline{a/\delta^2} = \bar{a}/\delta^2 = \frac{\text{area of hemisphere}}{\text{area of diametral plane}} = \frac{2\pi r^2}{\pi r^2} = 2 \qquad (69)$$

Introducing this result into equation (67) then gives

$$\bar{\lambda} = 2f/\beta\,. \qquad (70)$$

A method for measuring β has been described by Brooks and Purcell [7], but for present purposes the data on randomly packed uniform spheres, for which β can be computed, will suffice. For such a system, with n spheres per unit volume,

$$\beta = \frac{\text{area of spheres}}{\text{total volume}} = \frac{n\,.\,4\pi r^2}{\dfrac{n}{1-f}\,.\,\dfrac{4\pi r^3}{3}} = \frac{3(1-f)}{r} = \frac{6(1-f)}{d}, \qquad (71)$$

where d is the sphere diameter.

Then, introducing this result into equation (70), we obtain for packs of uniform spheres

$$\bar{\lambda} = \frac{f}{3(1-f)}\,.\,d; \qquad (72)$$

and

$$\bar{\lambda}^2 = \frac{f^2}{9(1-f)^2}\,.\,d^2\,. \qquad (73)$$

Comparison with experimental data.—In order to compare the value of $N_{\bar{\lambda}}$, corresponding to the characteristic length $\bar{\lambda}$, with N_d, corresponding to the sphere diameter d, we must first establish the relation between $N_{\bar{\lambda}}$ and N_d. This can be done by noting that, by definition,

$$N_{\bar{\lambda}}\bar{\lambda}^2 = N_d\, d^2 = k\,,$$

where k is the permeability of the system. Consequently

$$N_{\bar{\lambda}} = N_d\,(d^2/\bar{\lambda}^2)\,. \qquad (74)$$

Then, introducing the value of $d^2/\bar{\lambda}^2$ from equation (73) into equation (74) gives for a pack of uniform spheres

$$N_{\bar{\lambda}} = \frac{9(1-f)^2}{f^2}\,.\,N_d\,. \qquad (75)$$

The value of N_d for well-rounded quartz sands, screened to nearly uniform sizes, has been determined by the author's former research assistant, Jerry Conner. Using packs of different uniform sands with mean grain

diameters from 1.37×10^{-2} to 7.15×10^{-2} cm, Conner made seven independent determinations of N_d. The average value obtained was

$$\bar{N}_d = 6.0 \times 10^{-4}, \tag{76}$$

with individual values falling within the range between 5.3×10^{-4} and 6.7×10^{-4}.

Conner did not determine the porosity, but the average value of the porosity of randomly packed, uniform spherical glass beads found by Brooks and Purcell[7] was 0.37. Inserting this value of f, and Conner's value of N_d, into equation (75), then gives for the equivalent experimental value of $N_{\bar{\lambda}}$:

$$N_{\bar{\lambda}} = 26.2\, N_d = 1.57 \times 10^{-2}. \tag{77}$$

Comparing this experimental results with the approximate theoretical result of equation (63), it will be seen that

$$\frac{N_{\bar{\lambda}}\ \text{(theoretical)}}{N_{\bar{\lambda}}\ \text{(observed)}} = \frac{f/6}{1.57 \times 10^{-2}} = 4. \tag{78}$$

Since our object at the outset was merely to gain some insight into the nature of the shape-factor N occurring in Darcy's law, no particular concern is to be felt over the discrepancy in equation (78) between the observed and the theoretical values. All that this really indicates is that the system of averaging required should be better than the oversimplified one actually used. For a more complete analysis, account needs to be taken of the frequency distribution of the half gap-width λ, and also of the functional relation between $\bar{u}$ and λ. The fact that our approximate analysis yields a result in error by only a factor of 4 makes it appear promising that if account is taken of the variability of λ and of $\bar{u}$ as a function of λ, much better approximations may be obtainable.

Darcy's law for compressible fluids.—Our analysis thus far has been restricted to the flow of incompressible fluids for which the divergence term, $-(1/3)\,\mu\,\nabla\nabla\cdot\vec{v}$, could be eliminated from the Navier-Stokes equation. For the flow of a compressible fluid, this term must be retained, and with the flow parallel to the x-axis,

$$F_d = -\mu \int \left[\frac{4}{3}\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} + \frac{1}{3}\left(\frac{\partial^2 v}{\partial x \partial y} + \frac{\partial^2 w}{\partial x \partial z}\right)\right] dV, \tag{79}$$

of which the last two terms are equal to zero, leaving

$$F_d = -\mu \int \left(\frac{4}{3}\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) dV,$$

or

$$F_d = -2\mu\,[(2/3)\overline{(\partial^2 u/\partial x^2)} + \overline{(\partial^2 u/\partial y^2)}]\,f\Delta V. \tag{80}$$

Here, $\overline{\partial^2 u/\partial x^2}$ represents the gradient of the divergence of the velocity in the x-direction, or the rate of the fluid expansion. Should this term be of the same order of magnitude as $\overline{\partial^2 u/\partial y^2}$, and if the flow of a gas through the porous system is otherwise similar to that of a liquid, then the viscous resistance to a gas should be greater than that for a liquid of the same viscosity.

To compare the two terms $\overline{\partial^2 u/\partial x^2}$ and $\overline{\partial^2 u/\partial y^2}$, it will be noted that each is of the form: velocity/(length)^2. We have already seen that

$$\overline{\partial^2 u/\partial y^2} \cong -3\,\overline{u}/\overline{\lambda^2} \cong -q\bar{\lambda}^2,$$

indicating that the magnitude of this term is determined by the fact that large variations of u in the y-direction take place within the width of a single pore. Comparable variations of u in the x-direction, however, due to the expansion of the fluid, occur only in fairly large macroscopic distances. Consequently we may write

$$\overline{\partial^2 u/\partial x^2} \simeq \partial^2 q/\partial x^2 \simeq q/l^2 ,$$

where l is a macroscopic distance. The ratio of the two terms is accordingly

$$\frac{\overline{\partial^2 u/\partial x^2}}{\overline{\partial^2 u/\partial y^2}} \approx \frac{\bar{\lambda}^2}{l^2} . \tag{81}$$

Then, since $l^2 >> \bar{\lambda}^2$, it follows that the additional frictional drag caused by the divergence term is negligible, and this term may be deleted from the equation.

We conclude, therefore, from this approximate analysis, that Darcy's law in its differential form is the same for a gas as for a liquid, *provided that the flow behavior of a gas in small pore spaces, other than expansion, is similar to that of a liquid.*

It has been conclusively shown, however, by L. J. Klinkenberg [8] that the two flows are not similar, and that, in general, k_g, the permeability to gas based on the assumed validity of Darcy's law for gases, is not equal to k_e, the permeability to liquids; and, in fact, is not even a constant.

In the case of the flow of a liquid through small pores, the microscopic velocity $\vec{v}$ becomes zero at the fluid-solid boundary; for gas flow, on the contrary, there exists along the boundary a zone of slippage of thickness δ, which is proportional to the length of the mean-free path of the molecules. Consequently the gas velocity does not become zero at the boundaries, and the

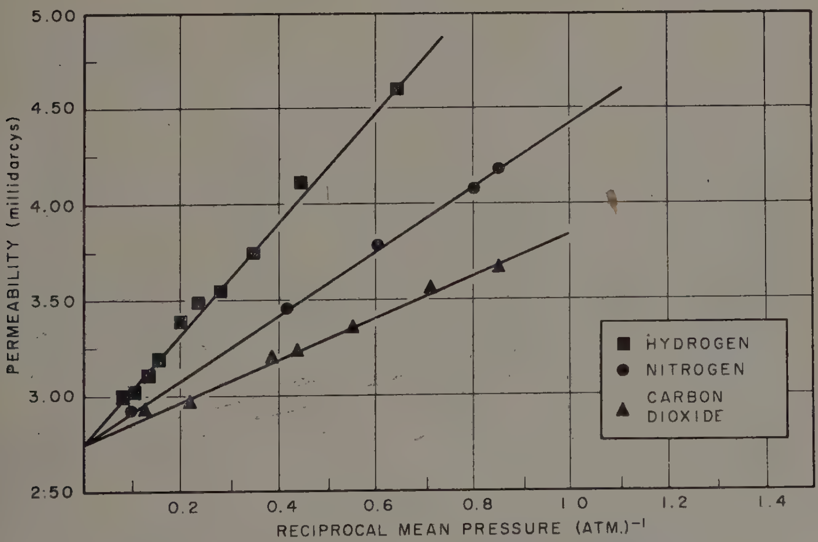

FIG. 7 — Variation as a function of $1/p$ of the apparent permeability of a given solid as determined by the different gases. The value of 2.75 millidarcys, as $(1/p)$ tends to 0, differs but slightly from that of 2.55 millidarcys obtained using a liquid (after L. J. Klinkenberg, *API Drilling and Production Practice*, 1941)

frictional resistance to the flow of gas is less than that for a liquid of the same viscosity and macroscopic velocity.

Since δ is proportional to the mean-free path, it is also approximately proportional to $1/p$. Consequently when the gas permeability, k_g, of a given porous solid is determined with the same gas at a number of different mean pressures, the resulting values of k_g, when plotted as a function of $1/p$, give a curve which is approximately linear with $1/p$. Moreover, different gases, having different mean-free paths, give curves of different slopes. The limiting value of k_g as $1/p \rightarrow 0$, or as p becomes very large, is also equal to k_l, the permeability obtained by means of a liquid (Fig. 7).

In view of this fact it is clear that, in general, the flow of gases through porous solids is not in accordance with Darcy's law. However, from Klinkenberg's data, at pressures greater than about 20 atmospheres (2×10^7 dynes/cm^2, or 300 psi), the value of k_g differs from k_l by less than 1 percent. Therefore, since most oil and gas reservoir pressures are much higher than this, it can be assumed that gases do obey Darcy's law under most reservoir conditions.

Field Equations of the Flow of Fluids Through Porous Solids

The establishment of Darcy's law provides a basis upon which we may now consider the field equations that must be satisfied by the flow of fluids through porous solids in general, three-dimensional space. This problem is complicated, however, by the fact that the fluids considered may be either of constant or of variable density; that one or several different fluids, either intermixed or segregated into separate macroscopic spaces, may be present simultaneously; and that all degrees of saturation of the space considered are possible.

The problem of dealing with such cases becomes tractable when we recognize that the behavior of each different fluid can be treated separately. Thus, for a specified fluid, there will exist at each point in space capable of being occupied by that fluid a macroscopic force intensity vector $\vec{E}$, defined as the force per unit mass that would act upon a macroscopic element of the fluid if placed at that point. In addition, if the fluid does occupy the space, its macroscopic flow rate at the given point will be indicated by the velocity vector $\vec{q}$, the volume of the fluid crossing unit area normal to the flow direction in unit time.

We shall thus have for each fluid two superposed fields, a field of force and a field of flow, each independently determinable. The equations describing the properties of each of these fields, and their mutual interrelations, comprise the field equations of the system; these in turn, in conjunction with the boundary conditions, determine the nature of the flow.

The field of force.—We have seen already that the force per unit mass is given by

$$\vec{E} = \vec{g} - (1/\rho) \operatorname{grad} p , \tag{15}$$

and the force per unit volume by

$$\vec{H} = \rho\vec{E} = \rho\vec{g} - \operatorname{grad} p . \tag{17}$$

Either of these force vectors could be used, and either is determinable from the other, but before choosing one in preference to the other, let us first

consider the properties of their respective fields, of which the most important for present purposes is whether or not the field has a potential. To simplify our analysis we will make the approximations that

$$\vec{g} = \text{const}\,, \tag{82}$$

and for chemically homogeneous liquids under the range of temperatures and pressures normally encountered in the earth to drillable depths,

$$\rho = \text{const}\,. \tag{83}$$

For gases, on the other hand, we shall have an equation of state

$$\rho = f(p, \mathrm{T}) \tag{84}$$

where T is the absolute temperature.

The vector $\vec{E}$ has a particular value at each point in space and the ensemble of such values comprises its vector field. The criterion of whether this field has a potential, that is to say, of whether

$$\vec{E} = -\,\text{grad}\,\Phi\,,$$

where Φ is a scalar field, is whether the field $\vec{E}$ is irrotational, which can be determined from its curl. From equation (15),

$$\begin{aligned}\text{curl}\,\vec{E} &= \text{curl}\,[\vec{g} - (1/\rho)\,\text{grad}\,p] \\ &= \nabla \times \vec{g} - \nabla \times [(1/\rho)\,\nabla p]\,.\end{aligned}$$

As is well known, the gravity field is irrotational even without the assumption that $\vec{g} = \text{const}$, so that $\nabla \times \vec{g} = 0$. Also

$$\begin{aligned}-\nabla \times [(1/\rho)\,\nabla p] &= -\nabla(1/\rho) \times \nabla p - (1/\rho)\,\nabla \times \nabla p \\ &= -\nabla(1/\rho) \times \nabla p\,.\end{aligned}$$

Consequently

$$\text{curl}\,\vec{E} = -\nabla(1/\rho) \times \nabla p = \nabla p \times \nabla(1/\rho)\,, \tag{85}$$

so that

$$\text{curl}\,\vec{E} = 0 \text{ when } \nabla p \times \nabla(1/\rho) = 0\,. \tag{86}$$

Therefore, in order for the field to be irrotational, and hence derivable from a scalar potential, it is necessary either that $\nabla p = 0$, corresponding to constant pressure, or $\nabla(1/\rho) = 0$, corresponding to constant density, or else that the vectors ∇p and $\nabla(1/\rho)$ be colinear, corresponding to a coincidence of the surfaces of equal density and equal pressure.

The second of these three cases is satisfied by a liquid of constant density, and the third by a gas whose density is a function of the pressure only, such as occurs under either isothermal or adiabatic conditions. For the general case, however, of a gas for which $\rho = f(p, \mathrm{T})$, and the surfaces of equal temperature do not coincide with those of equal pressure, then the surfaces $\rho = \text{const}$ will also not coincide with the surfaces $p = \text{const}$ and we shall have two intersecting families of surfaces, $(1/\rho) = \text{const}$, and $p = \text{const}$, for which equation (85) applies.

This is the condition corresponding to thermal convection, and the fluid will have a convective circulation in the direction that will tend to bring the surfaces of equal density into coincidence with those of equal pressure, with the less dense fluid uppermost.

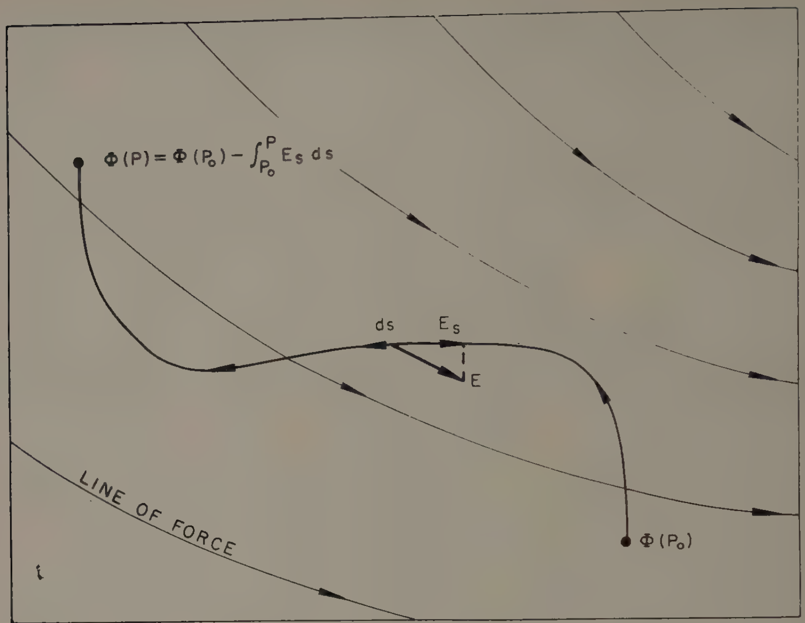

FIG. 8 — The potential Φ as a line integral of the field of force $\vec{E}$.

Hence, subject to the condition that either ρ = const, or $\rho = f(p)$,

$$\text{curl } \vec{E} = 0 \text{ and } \vec{E} = -\text{grad } \Phi\,. \tag{87}$$

The value of Φ at any arbitrary point P in space (Fig. 8) is then obtained by

$$\left.\begin{aligned}\Phi(P) &= \Phi(P_0) - \int_{P_0}^{P} E_s\, ds \\ &= \Phi(P_0) - \int_{P_0}^{P} \left[g_s - (1/\rho)\frac{\partial p}{\partial s}\right] ds \\ &= \Phi(P_0) + \int_0^z g dz + \int_{p_0}^{p} \frac{dp}{\rho} = \Phi(P_0) + gz + \int_{p_0}^{p} \frac{dp}{\rho}\,,\end{aligned}\right\} \tag{88}$$

where the integral from P_0 to P is taken along any path s. Then by setting $\Phi(P_0) = 0$ when $z = 0$ and $p_0 = 1$ atmosphere, we obtain

$$\Phi(P) = gz + \int_0^p \frac{dp}{\rho}\,, \tag{89}$$

where p is now the gage pressure, or the absolute pressure less 1 atmosphere.

If the fluid is incompressible and chemically homogeneous, this reduces to the simpler form

$$\Phi(P) = gz + p/\rho\,. \tag{90}$$

For this case, if a manometer is tapped into the system at the point P, the height h above the level $z = 0$ to which the liquid will rise, will be

$$h = z + p/\rho g\,, \tag{91}$$

from which it follows that

$$gh = gz + p/\rho = \Phi \,, \tag{92}$$

in agreement with our earlier definition of Φ in equation (14).

If the fluid is incompressible and chemically inhomogeneous, as in the case of water of variable salinity, the density ρ will not be a function of pressure only, and, in general, surfaces of constant density will not be parallel to surfaces of constant pressure. For such a system curl $\vec{E} \neq 0$, and no potential exists.

We thus see that, with the exception of cases of thermal convection, and of inhomogeneous liquids of variable density, that the fields $\vec{E}$ for both liquids and gases are irrotational and are derivable from a potential Φ. Since $\vec{E}$ is a force per unit of mass, then Φ is an energy per unit of mass, and represents the work required to transport the given fluid by a frictionless process along a prescribed (p, T)-path from a standard position and state to that of the point considered. Surfaces Φ = const are accordingly equipotential surfaces, or surfaces of constant energy of position, and the fluid will tend to flow from higher to lower potentials or energy levels.

The field of force per unit volume, $\vec{H}$, can be disposed of summarily. Since

$$\vec{H} = \rho \vec{g} - \text{grad}\, p \,,$$

then

$$\begin{aligned} \text{curl}\, \vec{H} &= \nabla \times (\rho \vec{g}) - \nabla \times \nabla p \\ &= \nabla \rho \times \vec{g} + \rho \nabla \times \vec{g} - \nabla \times \nabla p \,. \end{aligned}$$

But, since $\nabla \times \nabla p$ and $\nabla \times \vec{g}$ are each zero, then

$$\text{curl}\, \vec{H} = \nabla \rho \times \vec{g} \,. \tag{93}$$

This is zero only when ρ is constant or when the surfaces of constant density are horizontal. The last condition never occurs except when the motion is vertical. Hence, for motion in any direction other than vertical, the field of the vector $\vec{H}$ does not have a potential except when the density of the fluid is constant. For the special case of constant density,

$$\vec{H} = - \text{grad}\, \Pi \,, \tag{94}$$

where

$$\Pi = \rho \Phi = \rho g z + p \tag{95}$$

is the energy per unit volume of the fluid at any given point.

In view of the fact that the field $\vec{E}$ has a potential for both liquids and gases under the conditions specified above, whereas, in general, the field $\vec{H}$ has a potential only for the special case of liquids of constant density, then there is no advantage in using the latter in preference to the former, and henceforth it shall be dropped from further consideration.

The generality of the field of force, as herein defined, merits attention. The force vector $\vec{E}$ for any given fiuid not only has values in space occupied by that fluid, but also in any space capable of being occupied by the fluid. At a point in air, for example, the force $\vec{E}_w$ for water would be

$$\vec{E}_w = \vec{g} - (1/\rho_w)\, \text{grad p} \,,$$

and since, in air, grad p is $\rho_{air}\,\vec{g}$, then

$$\vec{E}_w = \frac{\rho_w - \rho_{air}}{\rho_w}\,\vec{g} \cong \vec{g}\,. \tag{96}$$

The field $\vec{E}$ for a given fluid thus extends throughout all space of continuous permeability. When several fluids are to be considered, then at each point in space there will be a different value of $\vec{E}$, for each separate fluid, given by:

$$\left.\begin{aligned} \vec{E}_1 &= \vec{g} - (1/\rho_1)\,\text{grad p}\,, \\ \vec{E}_2 &= \vec{g} - (1/\rho_2)\,\text{grad p}\,, \\ &\cdots\cdots\cdots \\ \vec{E}_n &= \vec{g} - (1/\rho_n)\,\text{grad p}\,. \end{aligned}\right\} \tag{97}$$

The vectors $\vec{E}$ for the separate fluids of different densities will differ among themselves, both in magnitude and direction, but will all fall in the same vertical plane, that defined by $\vec{g}$ and — grad p (Fig. 9).

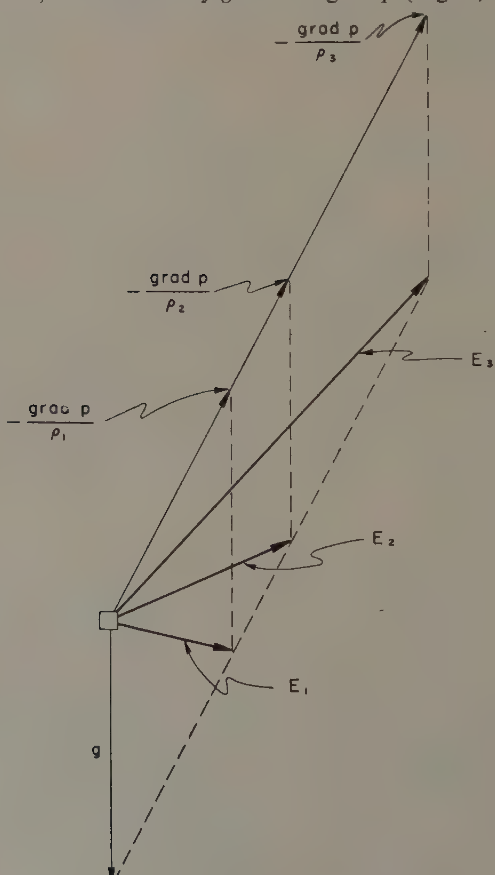

FIG 9 — Force vectors $\vec{E}$ at the same point corresponding to fluids of different densities.

Similarly the potentials of different fluids at the same point will be:

$$\left.\begin{aligned}\Phi_1 &= gz + \int (dp/\rho_1)\,,\\ \Phi_2 &= gz + \int (dp/\rho_2)\,,\\ &\cdots\cdots\cdots\cdots\\ \Phi_n &= gz + \int (dp/\rho_n)\,,\end{aligned}\right\} \tag{98}$$

where ρ_1, ρ_2, ... ρ_n are the variable densities of the separate fluids.

Since the equipotential surfaces for the separate fluids must be normal to the respective vectors $\vec{E}$, then it follows that the equipotential surfaces of different fluids passing through a given point will not be parallel to one another, although they will all intersect along a line normal to the $(\vec{g}, -\operatorname{grad} p)$-plane.

The field of flow.—We have already defined the flow vector $\vec{q}$ for a space which is entirely filled with a single fluid. For a space which is incompletely filled with a single fluid, or is occupied by two or more intermixed fluids, then there will be two or more superposed flow fields, not in general in the same direction, and a separate value of $\vec{q}$ for each separate fluid.

The principal condition which must be satisfied by the field of flow independently of the field of force is that it must be in accord with the principle of the conservation of mass. Thus, if a closed surface S, fixed with respect to the porous solid, is inscribed within the field of flow, then the total net outward mass flux of any given fluid in unit time will be equal to the diminution of the mass of that fluid enclosed by S, assuming that processes which create or consume the given fluid are forbidden.

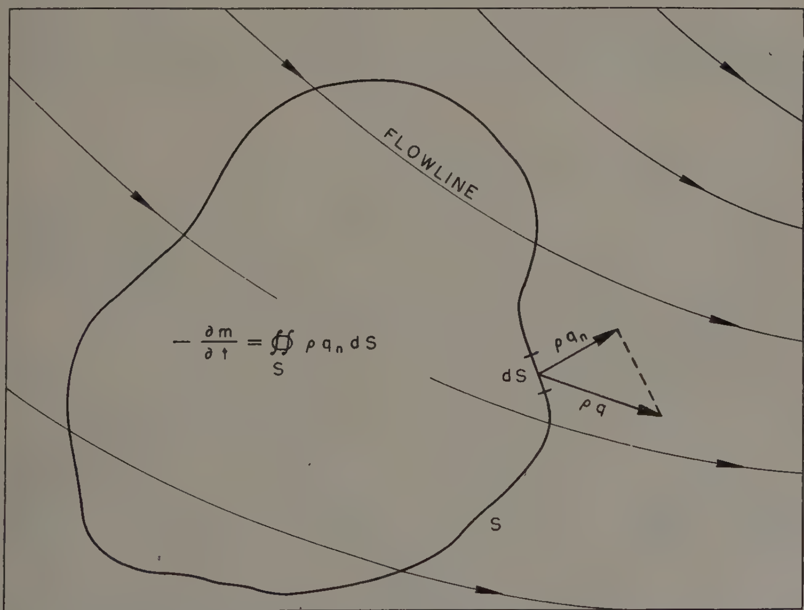

FIG 10 — Mass discharge through a fixed closed-surface S as the basis for the equation of continuity.

This condition is expressed by

$$\iint_S \rho q_n \, dS = -\partial m/\partial t \,, \tag{99}$$

where q_n is the outward-directed normal component of $\vec{q}$, and m the mass enclosed (Fig. 10).

In the case of a space completely saturated with the given fluid, by dividing the integral (99) by the volume V and then letting V tend to zero, we obtain

$$\operatorname{div} \rho\vec{q} = \lim_{V \to 0} \frac{1}{V} \iint_S \rho q_n \, dS = -\frac{1}{V} \frac{\partial m}{\partial t} \,, \tag{100}$$

which is the rate of loss of mass per unit macroscopic volume at a given point. Then, since

$$\partial m/\partial t = f V (\partial \rho/\partial t) \,,$$

equation (100) becomes

$$\operatorname{div} \rho\vec{q} = -f(\partial \rho/\partial t) \,, \tag{101}$$

which is the so-called «equation of continuity» of the flow. If the motion is steady, then $\partial\rho/\partial t$ is zero, and the equation simplifies to

$$\operatorname{div} \rho\vec{q} = 0 \,. \tag{102}$$

When this is expanded it becomes

$$\operatorname{div} \rho\vec{q} = \nabla \cdot \rho\vec{q} = \nabla\rho \cdot \vec{q} + \rho\nabla \cdot \vec{q} = 0 \,,$$

and when ρ is constant over space, corresponding to the flow of a homogeneous liquid, $\nabla\rho = 0$, and

$$\operatorname{div} \vec{q} = 0 \,. \tag{103}$$

Relation between field of force and field of flow.—In a space completely saturated by a single fluid, the field of flow and the field of force are linked together by Darcy's law

$$\vec{q} = \sigma\vec{E} \,, \tag{104}$$

which, in those cases for which curl $\vec{E} = 0$, becomes

$$\vec{q} = -\sigma \operatorname{grad} \Phi \,. \tag{105}$$

If the solid is isotropic with respect to permeability, the conductivity σ is a scalar and the flowlines and the lines of force will coincide; if the solid is anisotropic, σ will be a tensor and $\vec{E}$ and $\vec{q}$ will then differ somewhat in direction except when parallel to the principal axes of the tensor.

Limiting our discussion to isotropic systems, by taking the curl of equation (105), we obtain

$$\operatorname{curl} \vec{q} = -\nabla \times \sigma\nabla\Phi = -\nabla\sigma \times \nabla\Phi - \sigma\nabla \times \nabla\Phi \,.$$

Then, since the last term to the right is zero, this becomes

$$\operatorname{curl} \vec{q} = -\nabla\sigma \times \nabla\Phi \,, \tag{106}$$

which is zero only when σ is constant throughout the field of flow. Therefore, in general,

$$\operatorname{curl} \vec{q} \neq 0 \,, \tag{107}$$

and this circumstance precludes the derivation of the flow field from an assumed velocity potential, for, with the exception of the flow of a fluid of constant

density and viscosity in a space of constant permeability, no such function exists.

The flow of a given fluid through a porous solid incompletely saturated with that fluid is equivalent to flow through a solid of reduced permeability, because the space available to the flow diminishes as the saturation decreases. For saturations greater than some critical minimum value, the flow obeys Darcy's law subject to the permeability having this reduced, or so-called, «relative-permeability» value. Thus, with two interspersed but immiscible fluids in the same macroscopic space,

$$\left.\begin{aligned} \vec{q}_1 &= \sigma_{r1}\,\vec{E}_1 = \sigma_{r1}\,[\vec{g} - (1/\rho_1)\,\mathrm{grad}\,p]\,, \\ \vec{q}_2 &= \sigma_{r2}\,\vec{E}_2 = \sigma_{r2}\,[\vec{g} - (1/\rho_2)\,\mathrm{gard}\,p]\,, \end{aligned}\right\} \qquad (108)$$

where σ_{r1} and σ_{r2} are the relative conductivities of the two fluids.

It will be noted that, except for vertical motion, $\vec{E}_1$ and $\vec{E}_2$ are not parallel. Consequently the two force fields and flow fields will be, in general, transverse to one another.

Pioneer work on relative permeability as a function of saturation was done on the single fluid, water, by L. A. Richards [9] in 1931. Subsequently studies of the simultaneous flow of two or more fluids were initiated by Wyckoff and Botset, [10] and by Hassler, Rice and Leeman [11] in 1936. Since that time many other such studies for the systems water-oil-gas have been published.

One flaw which has been common to most of these multifluid experiments has been that the experimental arrangements and their interpretation were usually based upon the premise that the flowlines of the various components are all parallel and in the direction — grad p. Since this is far from true, there is some question of the degree of reliability of the results of such experiments. Even so, the existing evidence indicates that for saturations greater than some critical minimum the flow obeys Darcy's law in the form given in equations (108). For saturations less than this limit, there should still be a general drift of discontinuous fluid elements in the direction of the field $\vec{E}$, but with, as yet, no well-defined relationship between $\vec{q}$ and $\vec{E}$.

The migration of petroleum and natural gas, through an otherwise water-saturated underground environment, from an initial state of high dispersion to final positions of concentration and entrapment, constitutes an example of the latter kind.

Properties of the combined field of flow.—Although the present paper does not permit of their elaboration, practically everything that is now known concerning the flow of fluids in a porous-solid, three-dimensional space is deducible from the foregoing field equations. The equations for the flow of an incompressible fluid are of the same form as those for the steady conduction of electricity. Consequently the well-known solutions to the electrical equations can be applied directly to analogous situations in fluid flow. The unsteady flow of a compressible fluid, when the force field is irrotational, is closely analogous to the unsteady conduction of heat, although somewhat more complex, and the solutions of the heat equations can be adapted with some modification to the analogous fluid-flow problems.

The fluid phenomena which are unique and have no counterpart in other more familiar field theory are those involving multiple fluids. If we consider the flow of two immiscible fluids of unequal densities, such as water and oil,

interspersed in the same macroscopic space, we have seen from equation (97) that, in general, the force fields $\vec{E}_1$ and $\vec{E}_2$ of the two fluids will lie in the same vertical plane, but will have divergent directions, that for the less dense fluid being upward with respect to that for the more dense. As a consequence the two fluids will drift in these respective directions, and, in response to suitable impermeable barriers, will tend to become completely segregated.

Once this segregation is achieved, for any further steady motion of either of the fluids, the interface will appear as an impermeable barrier. Across this interface, neglecting minor pressure differences due to capillarity, the pressure in the two fluids must be the same. Then, in case neither of the fluids is in motion, the interface will have to be horizontal, with the less dense fluid uppermost, since this is the only inclination of the surface for which the pressures on opposite sides can be the same.

When either or both of the fluids is in motion, however, in a nonvertical direction, the vectors $(1/\rho_1)\,\mathrm{grad}\,p$ and $(1/\rho_2)\,\mathrm{grad}\,p$ will be inclined from the vertical, and the corresponding equipressure surfaces will be inclined from the horizontal by the same amounts. At the interface every equipressure surface in one system must match that of the same value in the other.

At the same time, the flowlines in each system must be parallel to the interface so that the $(\vec{E}_1, \vec{E}_2)$-plane must be tangent to the interface. Consequently the interfacial surface and the flow patterns in the two systems must mutually adjust themselves until these conditions are simultaneously satisfied before a steady state of flow becomes possible.

Of particular interest is the special case of this general situation wherein one of the two fluids is in motion and the other is completely static. In this case the equipressure surfaces in the static fluid are horizontal and surfaces equal difference of pressure are equally spaced; whereas, in the moving fluid the equipressure surfaces are inclined downward in the direction of the horizontal component of the flow. Their lines of intersection at the interface must accordingly be horizontal. Also, since the vector grad p lies in the same vertical plane as the vector $\vec{E}$ of the flowing fluid, it follows that the horizontal component of the flow direction must be parallel to the direction of steepest slope of the interface.

If we consider a vertical plane parallel to the flow direction and perpendicular to the interface, then in this plane the surfaces of constant pressure will appear as lines of constant pressure, inclined in the flowing fluid and refracting into the horizontal across the interface in the static fluid. Let the flowing fluid be denoted by the subscript 1 and the static fluid by the subscript 2. Then along the interface in the direction of the flow

$$(\partial p/\partial s)_1 = (\partial p/\partial s)_2 \,. \tag{109}$$

In the static fluid

$$(\partial p/\partial s)_2 = -\,\rho_2 g \sin\theta \,, \tag{110}$$

and consequently depends only upon the density ρ_2 and the angle of slope θ, where θ is positive when the slope is upward in the flow direction.

In the flowing fluid, since

$$E = g_s - (1/\rho_1)(\partial p/\partial s)_1 \,,$$

then

$$\left.\begin{aligned}(\partial p/\partial s)_1 &= \rho_1 g_s - \rho_1 E \\ &= -\,\rho_1 g \sin\theta + \rho_1 \left|\mathrm{grad}\,\Phi_1\right| .\end{aligned}\right\} \tag{111}$$

Equating (110) and (111) then gives

$$\rho_2 g \sin \theta = \rho_1 g \sin \theta - \rho_1 |\text{grad } \Phi_1|,$$

which, when solved for sin θ, becomes

$$\sin \theta = \frac{1}{g} \frac{\rho_1}{\rho_2 - \rho_1} |\text{grad } \Phi_1|. \tag{112}$$

If the less dense fluid is flowing (Fig. 11*a*), $\rho_2 > \rho_1$, the term to the right will be positive, and the interface will tilt upward; if the more dense fluid is flowing (Fig. 11*b*), $\rho_1 > \rho_2$, the term to the right will be negative, and the interface will tilt downward.—

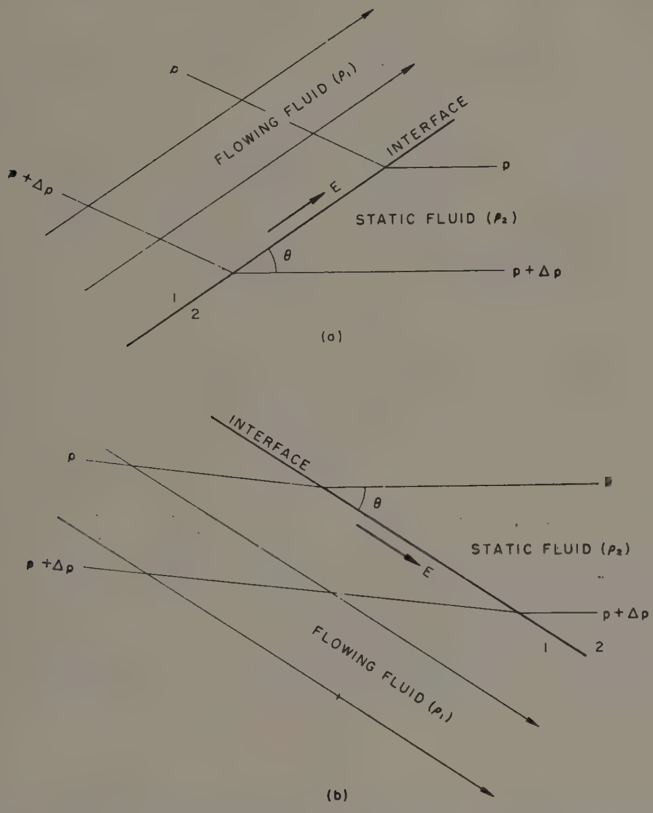

FIG. 11 — Interfaces between flowing and static fluids of unequal densities.
(*a*) Upward tilt in the flow direction when less dense fluid is flowing.
(*b*) Downward tilt when more dense fluid flows.

Equation (112), derived earlier by Hubbert, [12, 13] is the fundamental equation pertaining to fresh -water — salt-water relations along shore lines, and to oil-water and oil-gas interfacial relations such as water or gas coning during oil and gas exploitation. It is also the basic equation governing the underground positions of oil and gas entrapment. If the water is static, accumulations of petroleum or of natural gas will occur beneath

downwardly concave impermeable barriers, with the oil-water or gas-water interface horizontal. If the water is in motion, as often is the case, the oil-water or gas-water interface will be inclined, and oil or gas may be trapped beneath structures completely unclosed in the hydrostatic sense.

RÉSUMÉ

In paying our respects to M. Henry Darcy on this centennial occasion, we stated at the outset that it should be our endeavor:

1. To show unequivocally what Darcy himself did and stated with respect to the relationship which now bears his name, and to give his results a more general, but still equivalent, physical formulation.
2. To derive Darcy's law directly from the fundamental Navier-Stokes equation of motion of viscous fluids.
3. To develop, in at least their primitive forms, the principal field equations of the flow of fluids through porous solids.

These objectives have now been accomplished, and the result is that, despite a number of troublesome complexities such as those arising from thermal convection and from waters of variable salinity, the field theory of the flow of underground fluids, is capable of being brought into the same kind of a comprehensive unification as that already achieved for the more familiar phenomena of electrical and thermal conduction.

In closing it is pertinent to reiterate that Darcy's empirical formulation:

$$\vec{q} = K\,(h_1 - h_2)/l\,,$$

which, as we have seen is valid for flow of a homogeneous liquid in any direction, is physically equivalent to the expressions:

$$\vec{q} = -(Nd^2)(\rho/\mu)\,g\,\text{grad}\,h\,,$$

$$\vec{q} = +(Nd^2)(\rho/\mu)[\vec{g} - (1/\rho)\,\text{grad}\,p]\,.$$

From these it follows that fluids can and do flow in any direction whatever with respect to that of the pressure gradient.

So far as we have seen no error of any kind with respect to the flow of fluids through porous solids can be attributed to Henry Darcy; and the errors which have been alleged by various authors during recent years appear on closer inspection to have been those committed by the authors themselves.

*

Acknowledgments.—Since this paper is the result of some twenty years of intermittent reflection upon the phenomena encompassed by Darcy's law, the author is indebted to many people whose oral or written discussions of various aspects of the problems involved have contributed to his own understanding of them. He is particularly indebted, however, to his current and recent colleagues who have been of direct assistance in the present study. These include: Jerry Conner who made an extensive series of experiments verifying the author's earlier theoretical deductions; David G. Willis who, in addition to reading critically the manuscript and assisting in the design of the illustrations, has been of invaluable assistance in the clarification of many difficult points; R. L. Chuoke and A. S. Ginzbarg who gave important mathematical assistance; and R. H. Nanz who made an extensive series of

measurements of the gap-widths along linear traverses through randomly packed uniform spheres.

Any errors, however, are the authors own.

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LES RECHERCHES DE TECHNIQUE PLUVIOMÉTRIQUE DE LA STATION CENTRALE SUISSE DE MÉTÉOROLOGIE, M. Z. A. ZURICH (complément)

RÉSULTATS OBTENUS DU CHAMP D'ESSAIS DU SENTIER (JURA VAUDOIS), 1941-1951)

par Paul-L. Mercanton, Lausanne

(Cf. P-V. de la session d'Oslo, août 1948: « Les recherches de technique pluviométrique », p. 131. Pour le détail voir: *Annales de la M. Z. A.*, 1938 1939, 1940, 1944

Délimité au large des versants, sur le fond horizontal de la Vallée de Joux, en amont du lac et à 1000 m d'altitude, ce terrain avait reçu à fin avril 1941, un ensemble d'engins pluviométriques étudiés déjà à Zurich quant à leur neutralité aérodynamique, mais qu'il convenait de soumettre encore à des essais critiques en plein vent, dans des conditions variées et sévères. A ce dernier égard, le climat de la Vallée, rigoureux en hiver, volontiers chaud à l'excès en été, assurait d'utiles expériences que la situation du champ, en travers des vents, humides, du secteur ouest à sud, garantissait pleinement aussi.

Les observations se sont poursuivies sans discontinuer durant dix années par les soins de Mr. le professeur Pierre Baud principalement, avec la collaboration du sussigné, pour la M. Z. A. L'équipement instrumental n'a guère changé durant cette décennie, laps de temps d'ailleurs nécessaire pour une comparaison pertinente des appareils. Les résultats ayant paru au complet dans les Annales de la M. Z. A., il suffira ici de les récapituler:

Les deux grands totalisateurs du modèle classique de la M. Z. A., (Maurer et Billwiller) occupant les extrémités de la diagonale du champ ont prouvé l'uniforme ventilation du terrain: les quantités d'eau recueillies par eux et conservées d'automne à automne n'ont différé entre elles que de quelques dixièmes pour cent (moyenne des dix ans: 0,45 %). Le modèle normal de la M. Z. A. a toujours recueilli un peu davantage d'eau que l'autre, dont l'écran était abaissé de 13 cm par rapport à celui du premier dont l'écran a son bord supérieur au niveau de l'ouverture pluviométrique. Les essais au tunnel avaient avéré pour cet écran ainsi abaissé un optimum de neutralité dans le vent régulier, mais en est-il de même dans le vent naturel? Dans le doute, et aucun des engins du champ ne pouvant être tenu pour parfait, on a pris en général comme base de comparaison numérique la moyenne des indications annuelles des deux totalisateurs. Bien entendu, on a réduit les indications de tous les engins du champ à l'ouverture pluviométrique classique de 200 cm². Notons enfin que le totalisateur à écran abaissé a emmagasiné en moyenne un peu moins de précipitations durant la saison neigeuse que le totalisateur normal, chez lequel l'écran, par sa position, favorise un peu l'entrée de la neige.

Le totalisateur ellipsoïdal à vidange mensuelle, qui s'était montré aérodynamiquement parfait au tunnel, n'a pas répondu à notre espoir d'en faire la base des comparaisons avec les autres appareils. Si pour la pluie, le revêtement de brosse métallique (l'« oursin »), a été certainement assez efficace, on n'a pu réaliser en hiver un chauffage rationnel de l'engin; le rejaillissement

des gouttes de la pluie ou le balayage des flocons de la neige dans l'ouverture réceptrice ont en effet toujours exagéré les quantités d'eau recueillie. Nous avons donc dû le disqualifier comme étalon. Il a mesuré en moyenne générale 6 % de plus que les grands totalisateurs. Le pluvio-totalisateur mensuel M.Z.A.-Mercanton qui avait fait de bonnes preuves à Zurich, n'a pas supporté les rigueurs de l'hiver jurassien (—25° C et davantage) et a dû très tôt être supprimé.

Le Hellmann normal a mesuré 2 % en moyenne générale de plus que les totalisateurs annuels. Durant quelques années, mais en saison chaude seulement, on a expérimenté le pluviomètre sphérique Haas-Lütschg, un globe métallique présentant au vent, quelle qu'en soit la direction, un ensemble de trous circulaires, symétriquement répartis, et faisant en projection sur le plan normal à la dite direction, la surface de réception de 200 cm². L'eau engagée dans les ouvertures est canalisée vers un réservoir commun. L'appareil est affecté à un très haut degré des défauts qui nous ont fait rebuter les observations de l'engin ellipsoïdal; il a toujours indiqué des hauteurs d'eau considérablement supérieures à celles des pluviomètres à ouverture horizontale, ce qui se conçoit d'emblée. La signification des mesures au H-L n'est d'ailleurs pas claire.

Un dernier dispositif mérite de retenir l'attention par ses résultats prometteurs : le pluviomètre Hellmann de haut modèle (60 cm), entouré d'un écran de tôle plein centré sur l'axe du pluviomètre et fait de deux troncs de cône circulaires, raccordés en façon de poulie ou de diabolo par leurs petites sections. Haut de 60 cm aussi, cet écran présente en haut et en bas, des ouvertures circulaires de 54 cm sans rebords et se rétrécissant graduellement jusqu'à 44 cm en son milieu. Ces ouvertures terminales sont rigoureusement dans les plans de la bouche et du fond du pluviomètre; l'ensemble est supporté par trois pieds équidistants. Les essais à la soufflerie ont démontré la neutralité excellente de cette construction.

La comparaison pendant plusieurs années au champ d'essais avec l'Hellmann nu a donné les résultats suivants : le pluviomètre à écran en poulie a emmagasiné en pluie 2 % de moins que l'autre; il a recueilli en neige 2 % de plus. D'autre part ici encore une différence par excès est apparue avec la moyenne des totalisateurs annuels. Cette différence persistant entre les engins de mesures journalières du champ et ceux à longue échéance est troublante : elle force à se demander si la couche classique de 2 mm d'huile, recouvrant la liqueur saline des totalisateurs, supprime bien toute évaporation ?

Les essais au champ donnent enfin un regain d'intérêt à une question posée antérieurement par l'auteur (Recherches de technique pluviométrique III, Annales 1939) : un pluviomètre, abrité du vent par un entourage même distant, d'arbres ou d'édifices élevés, ne reçoit-il pas trop d'eau météorique ? En effet, le pluviomètre Hellmann normal du Collège installé à 200 m de notre champ d'essais et à une quinzaine de mètres au-dessus, a décelé pour les 10 ans entiers, un excès de 3 % sur l'Hellmann du champ et de 5 % sur les totalisateurs annuels.

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